Computing Mitered Offset Curves Based on Straight Skeletons

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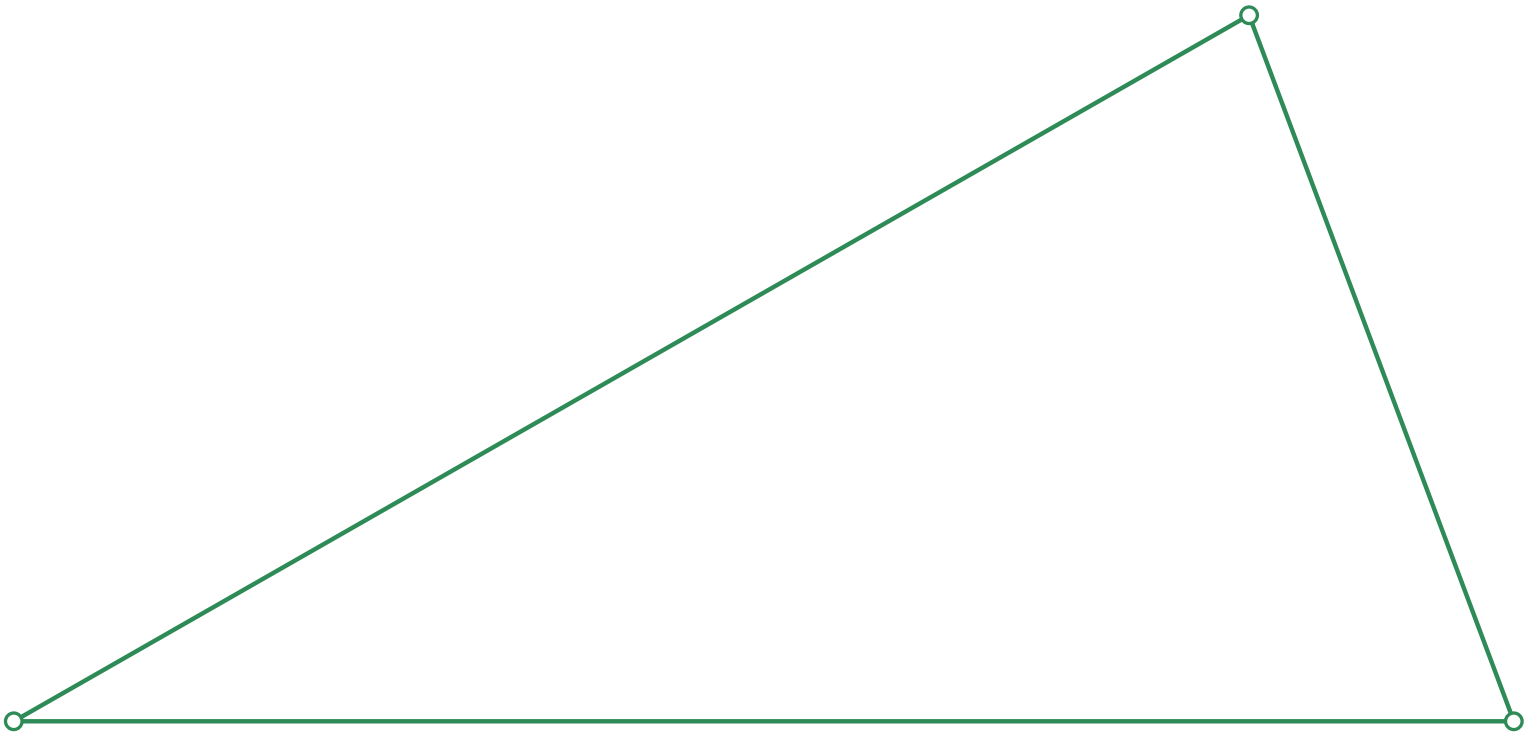
Straight Skeletons — Motivation

Aichholzer&Alberts&Aurenhammer&Gärtner (1995)



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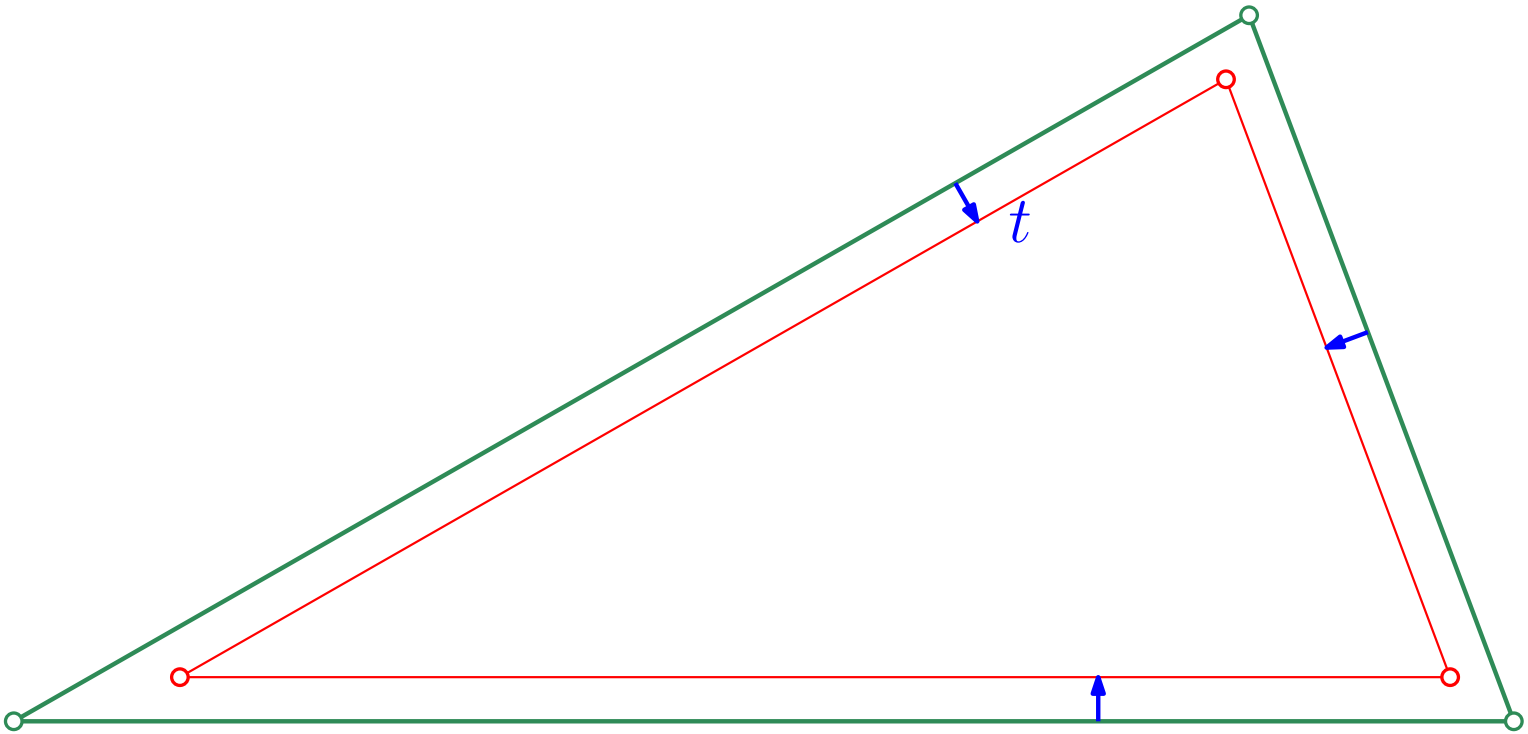

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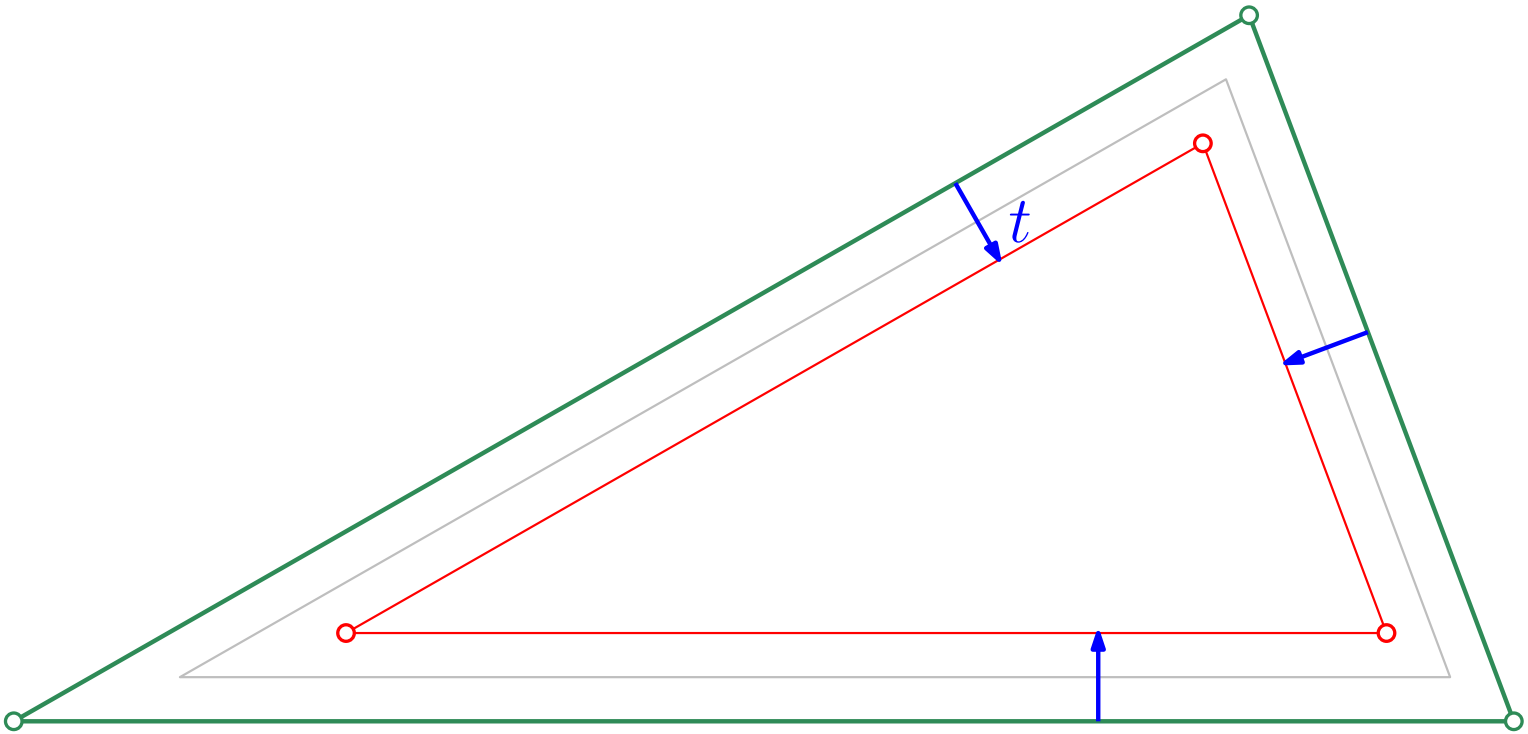
- Wavefront propagation with unit speed via continued offsetting: shrinking process, where offset distance t equals time.



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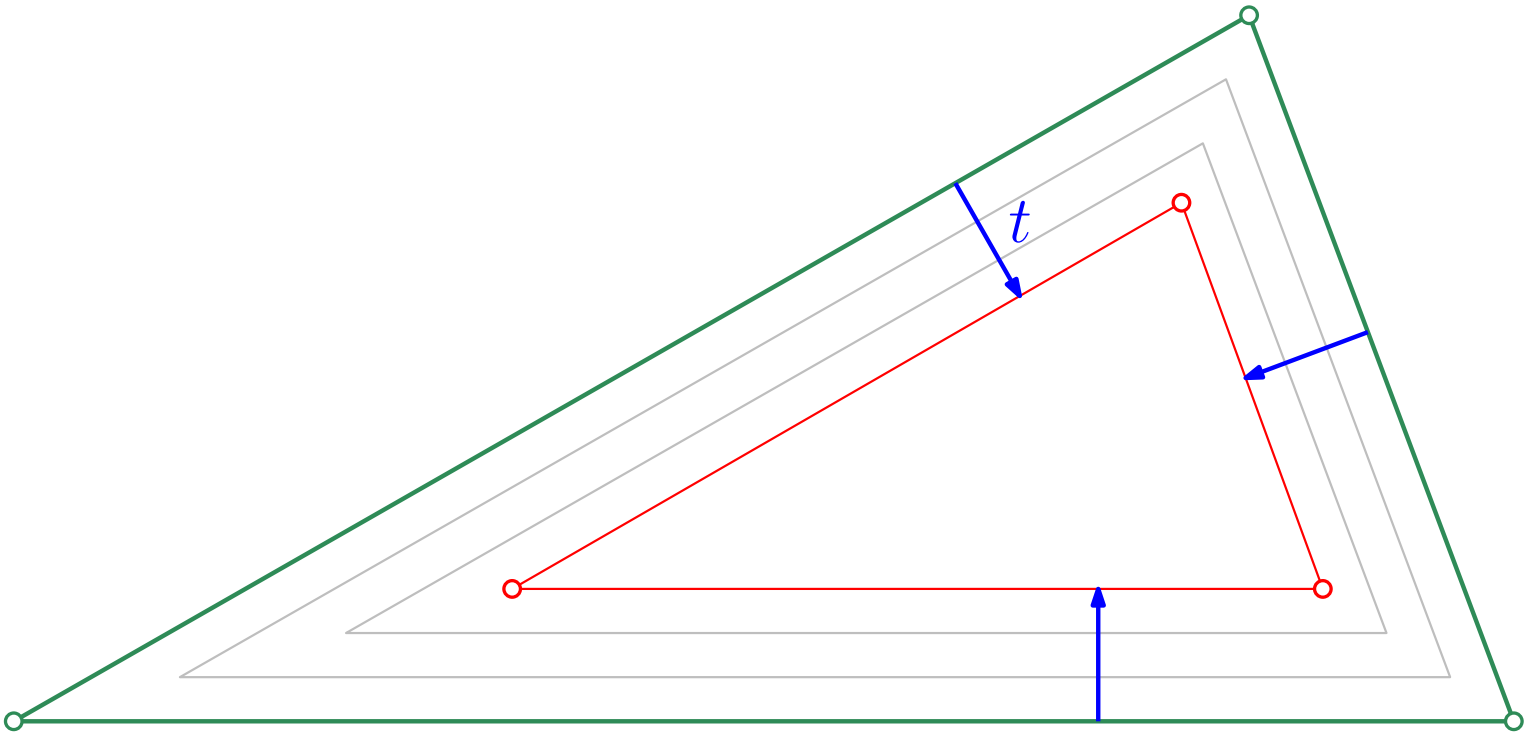

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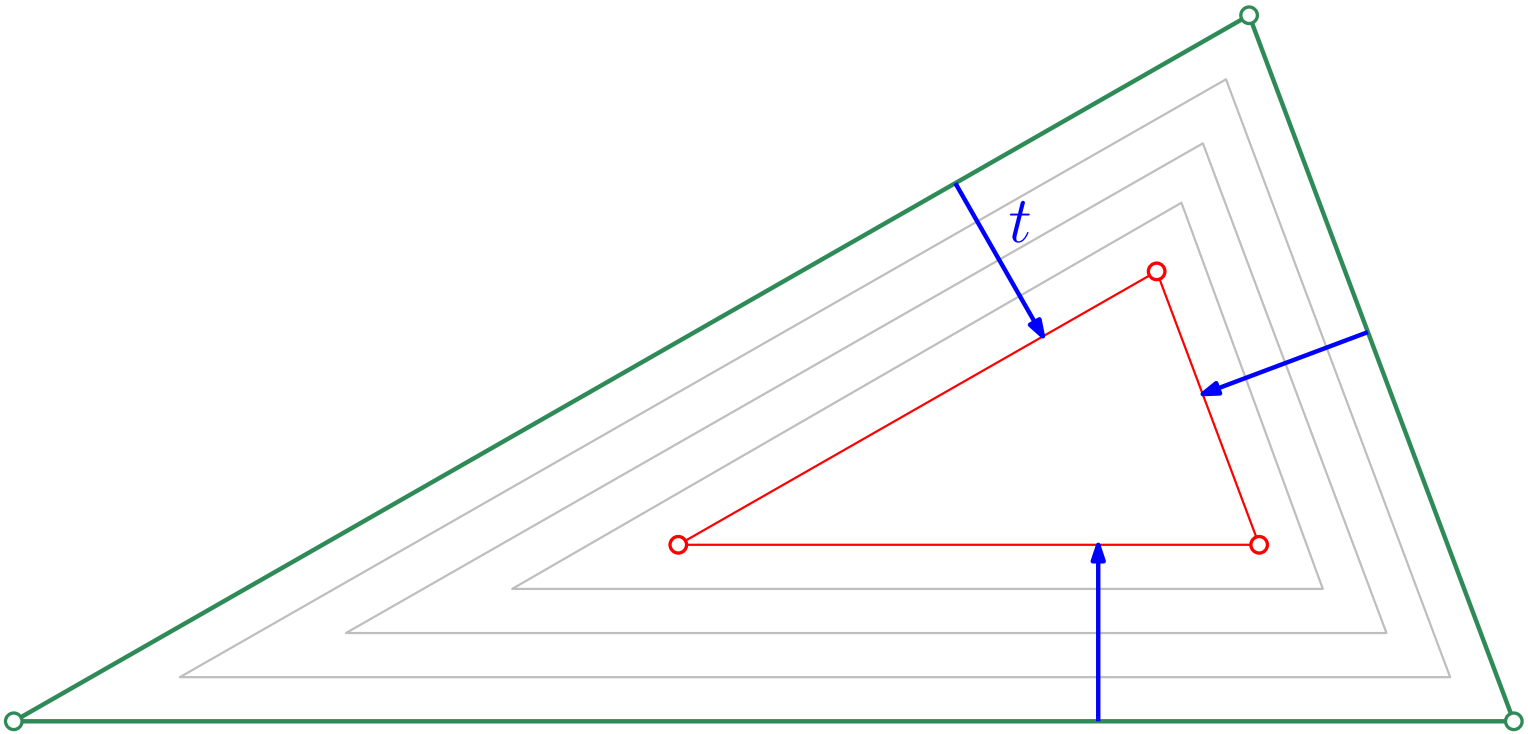

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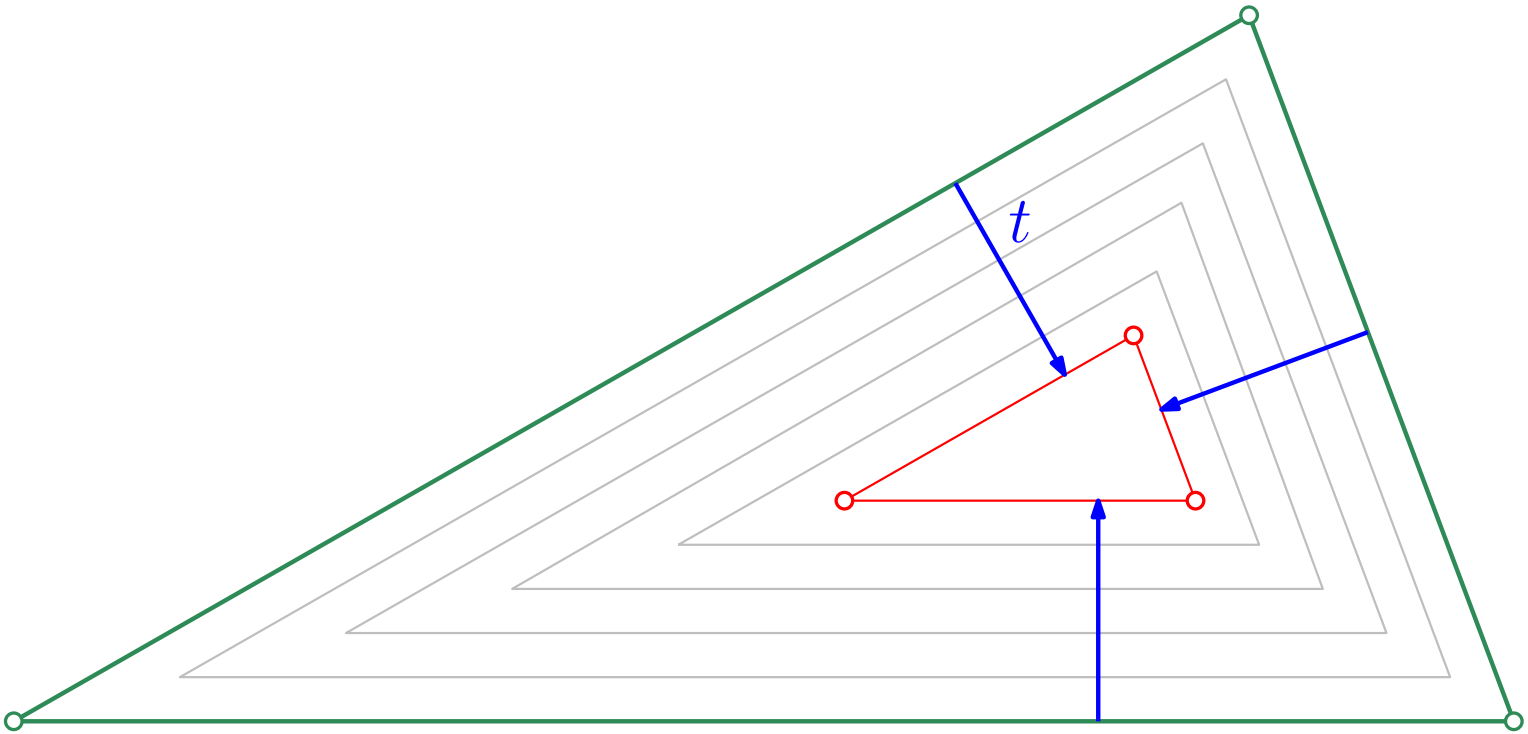

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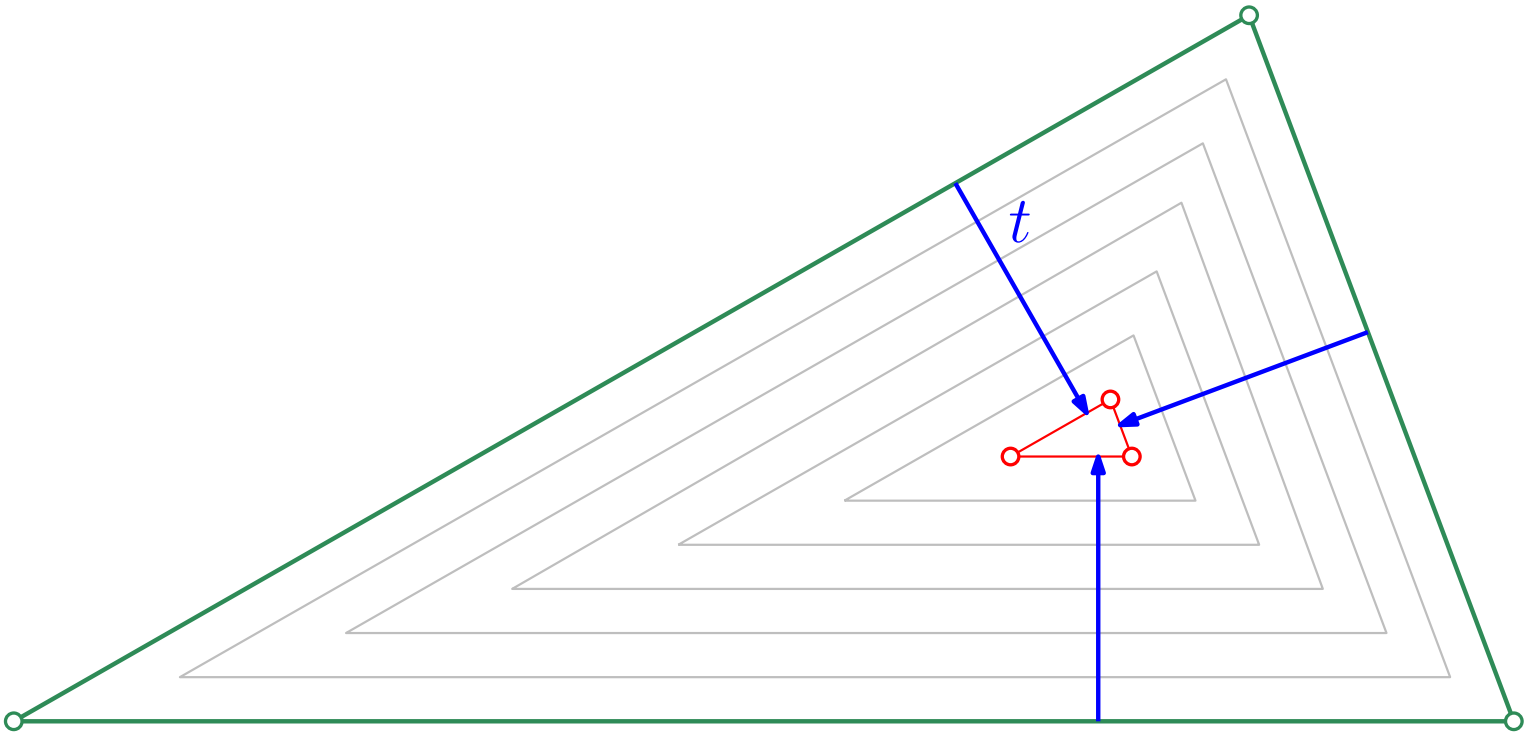

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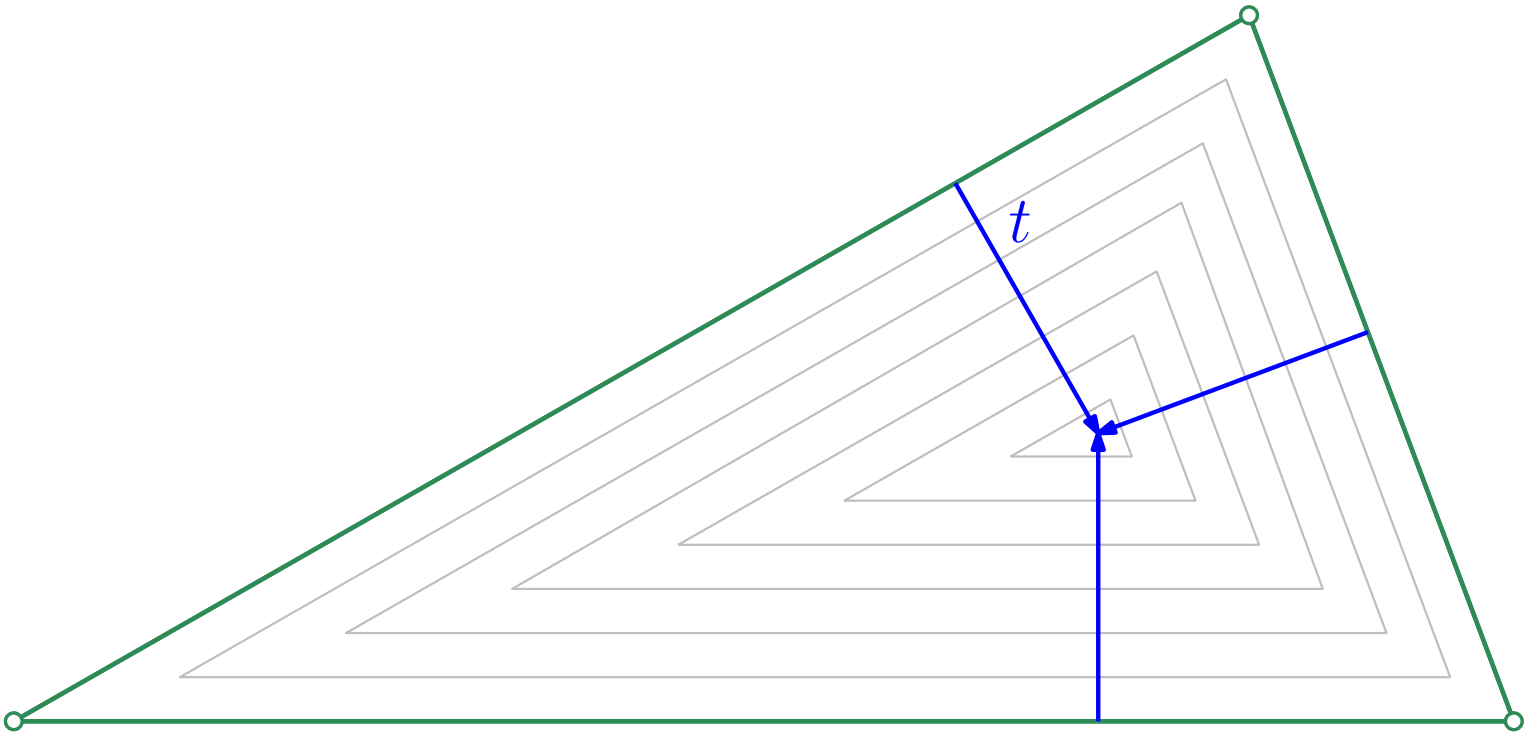

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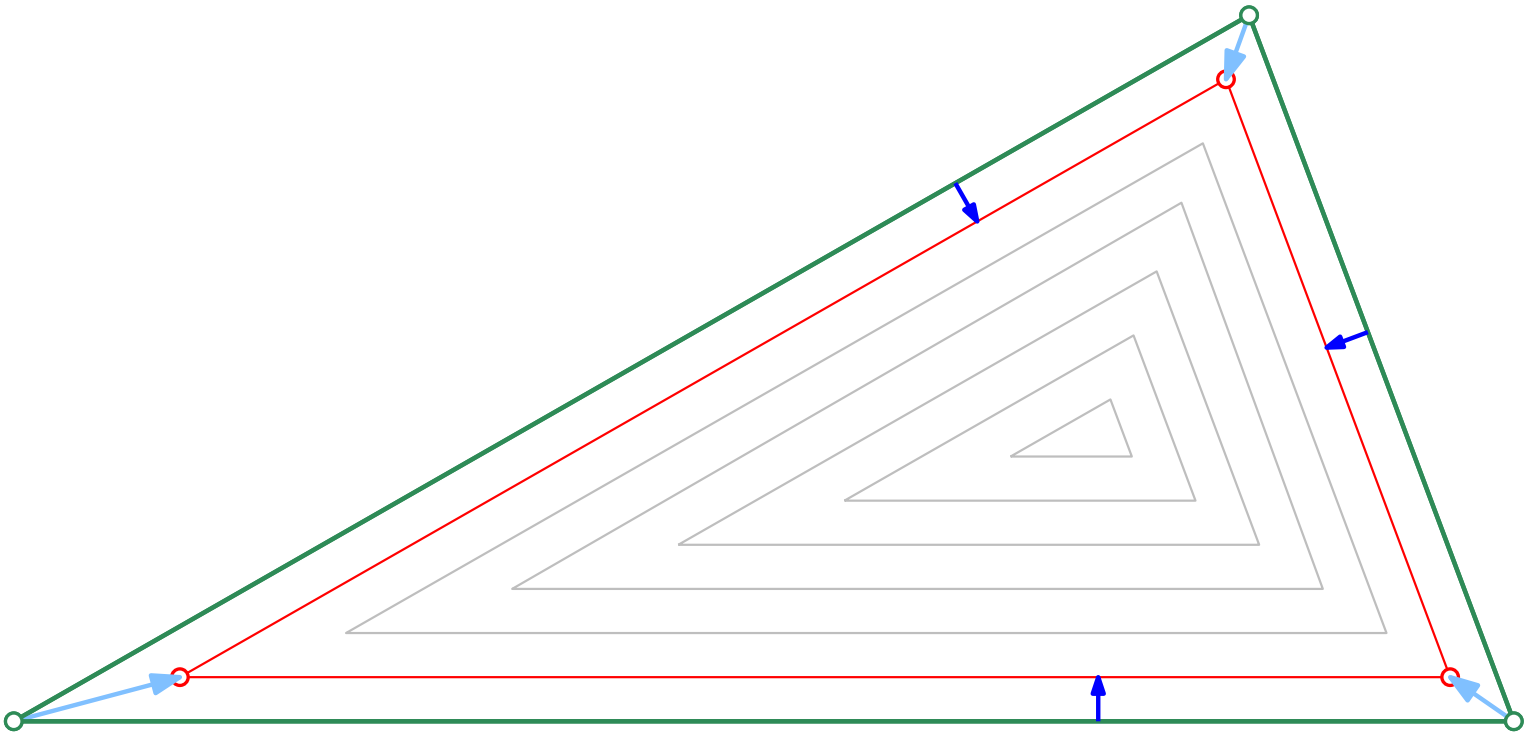

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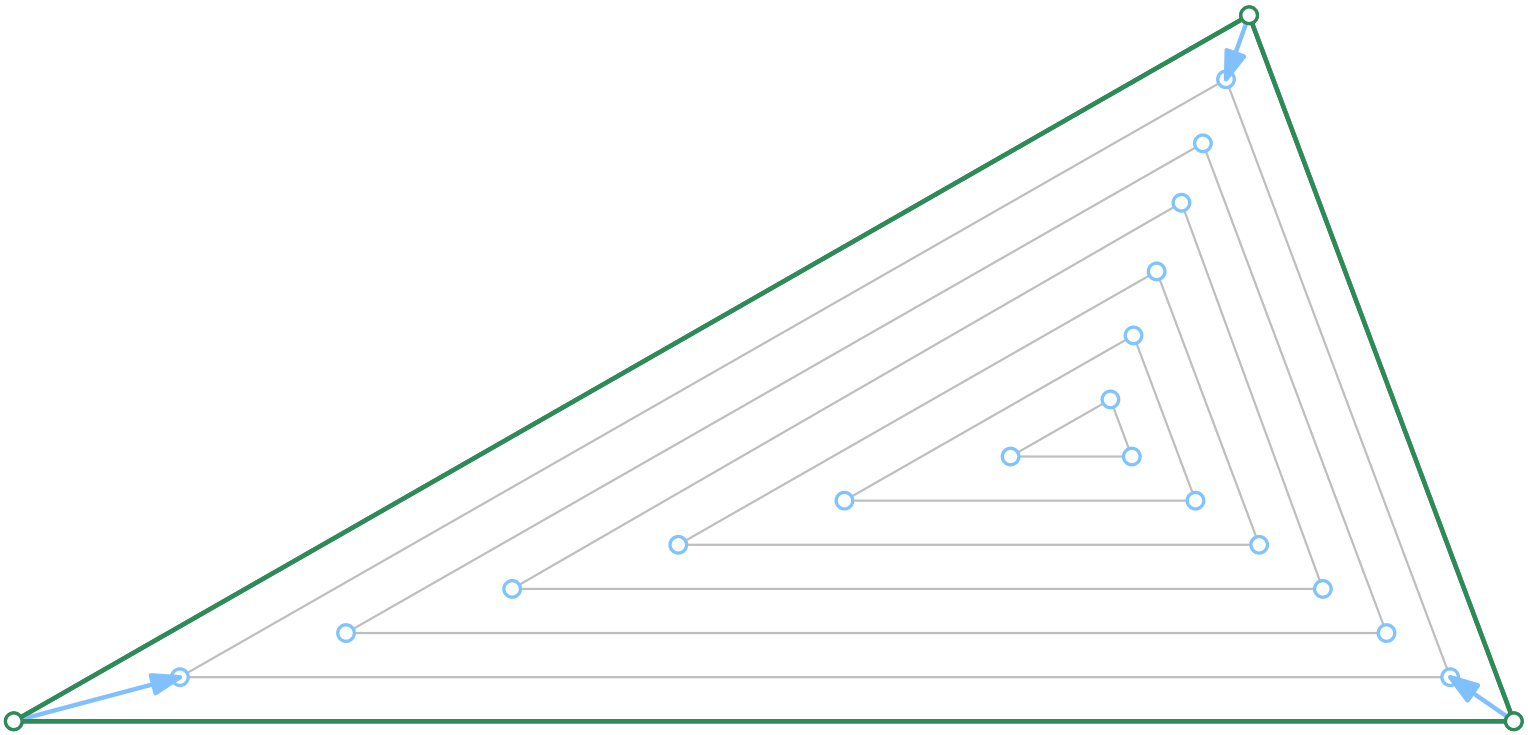

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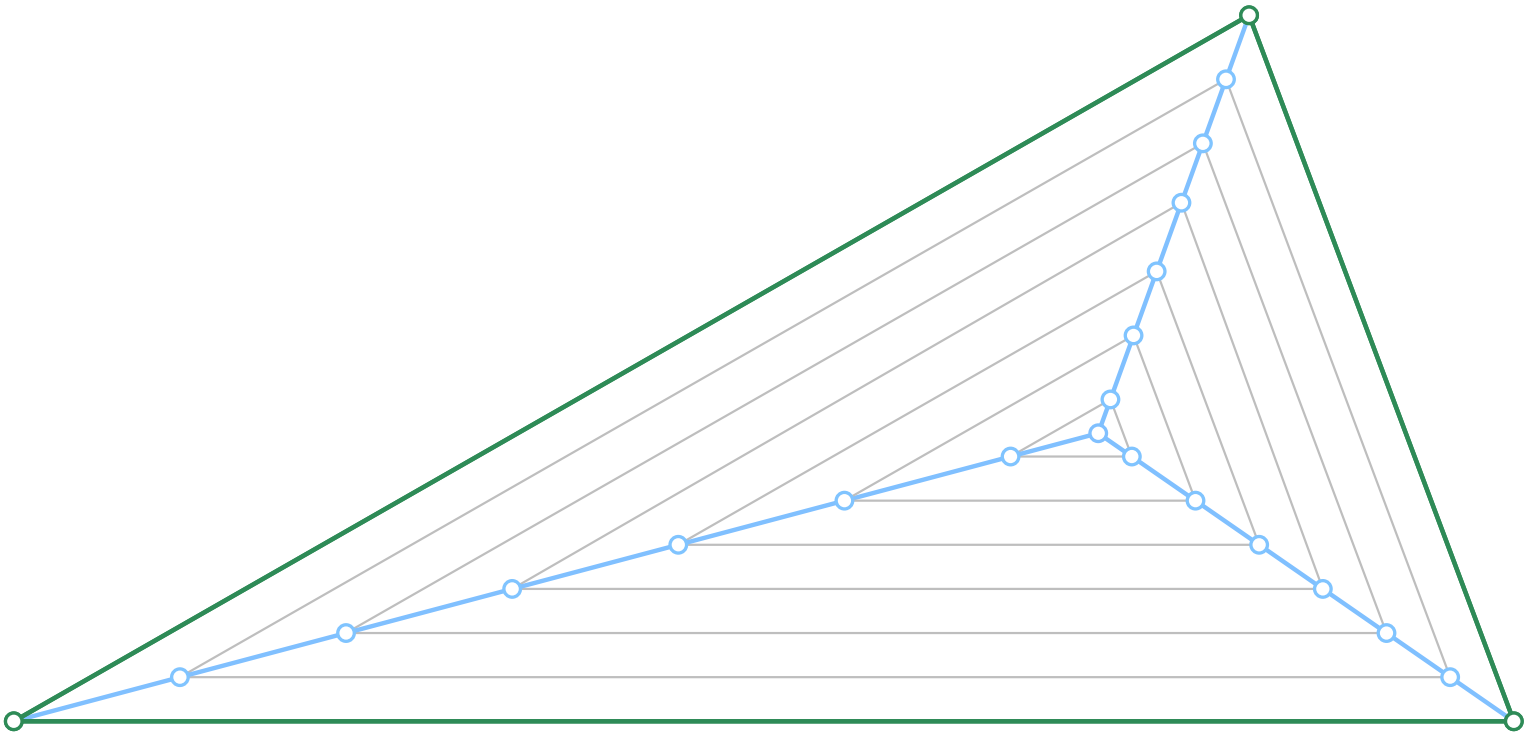
- Wavefront propagation with unit speed via continued offsetting: shrinking process, where offset distance t equals time.
- Straight skeleton $\mathcal{SK}(\mathcal{P})$ is union of traces of wavefront vertices.



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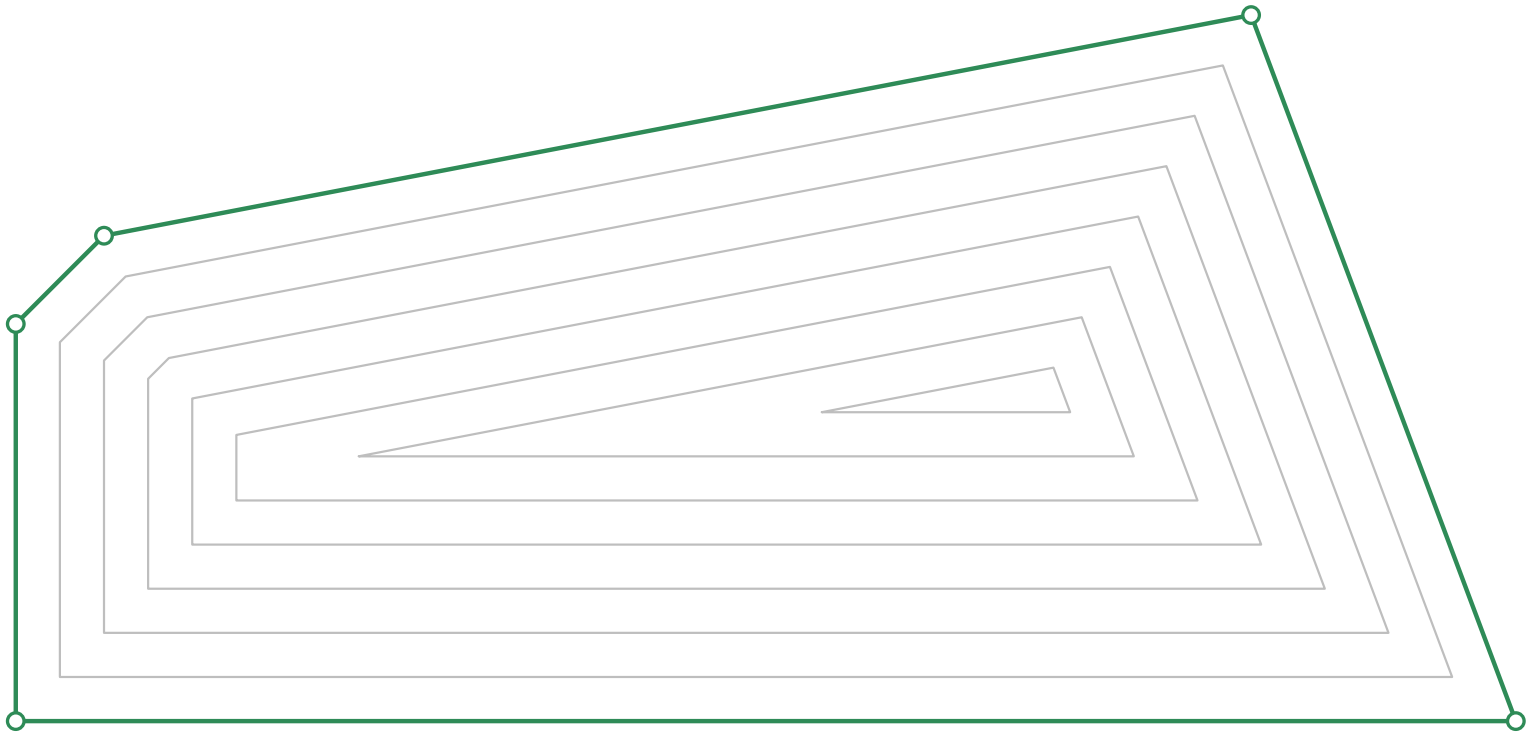
Change of Wavefront Topology

Edge event

- Wavefront topology changes over time.



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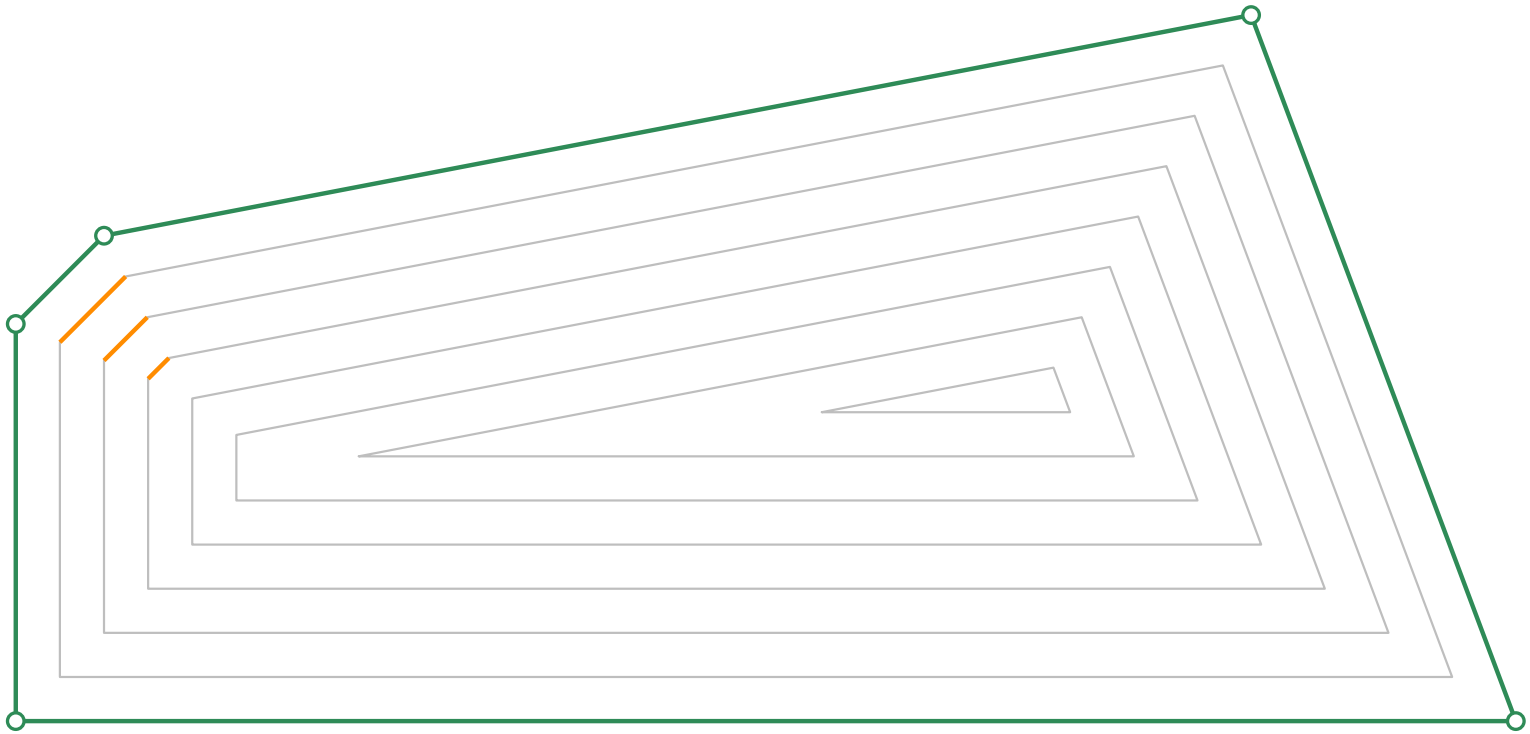

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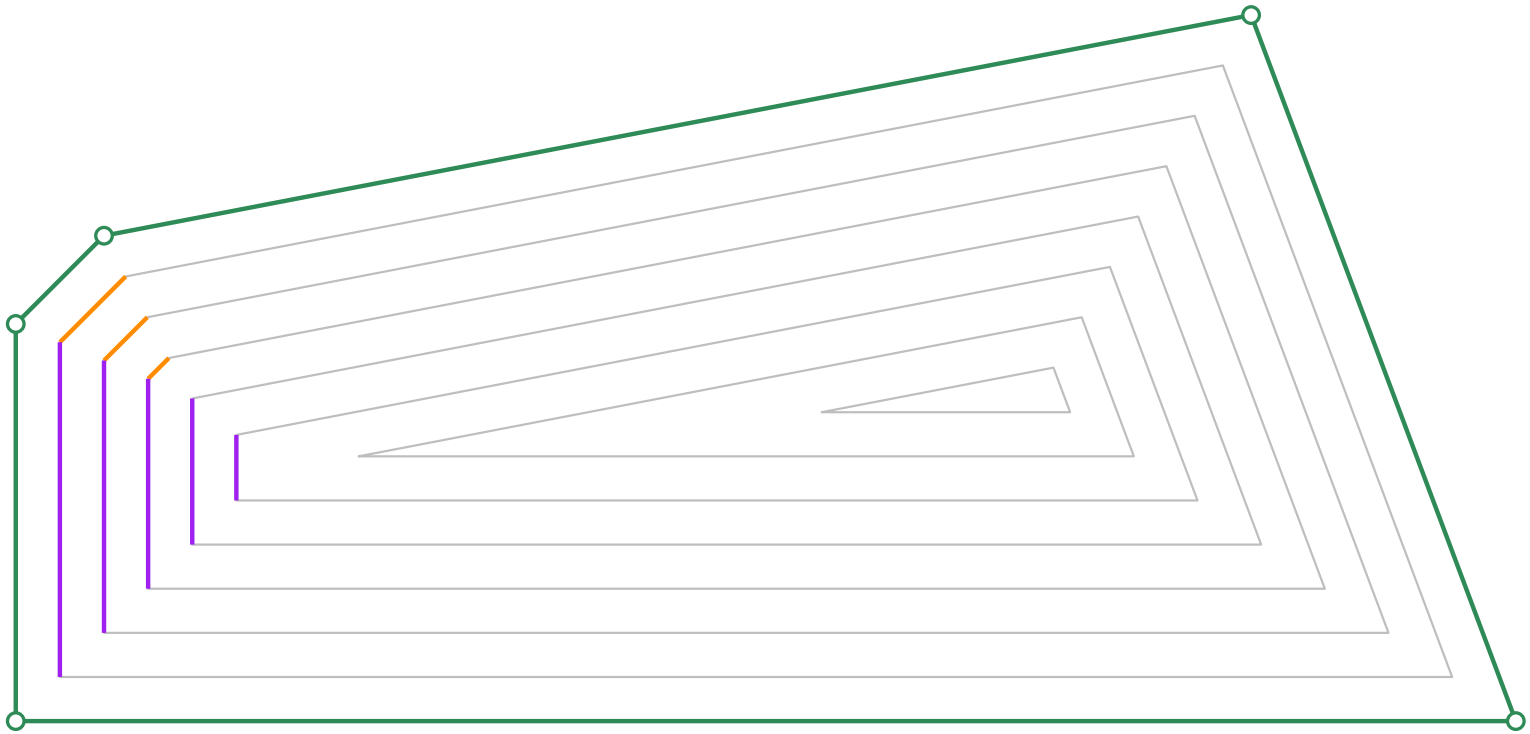

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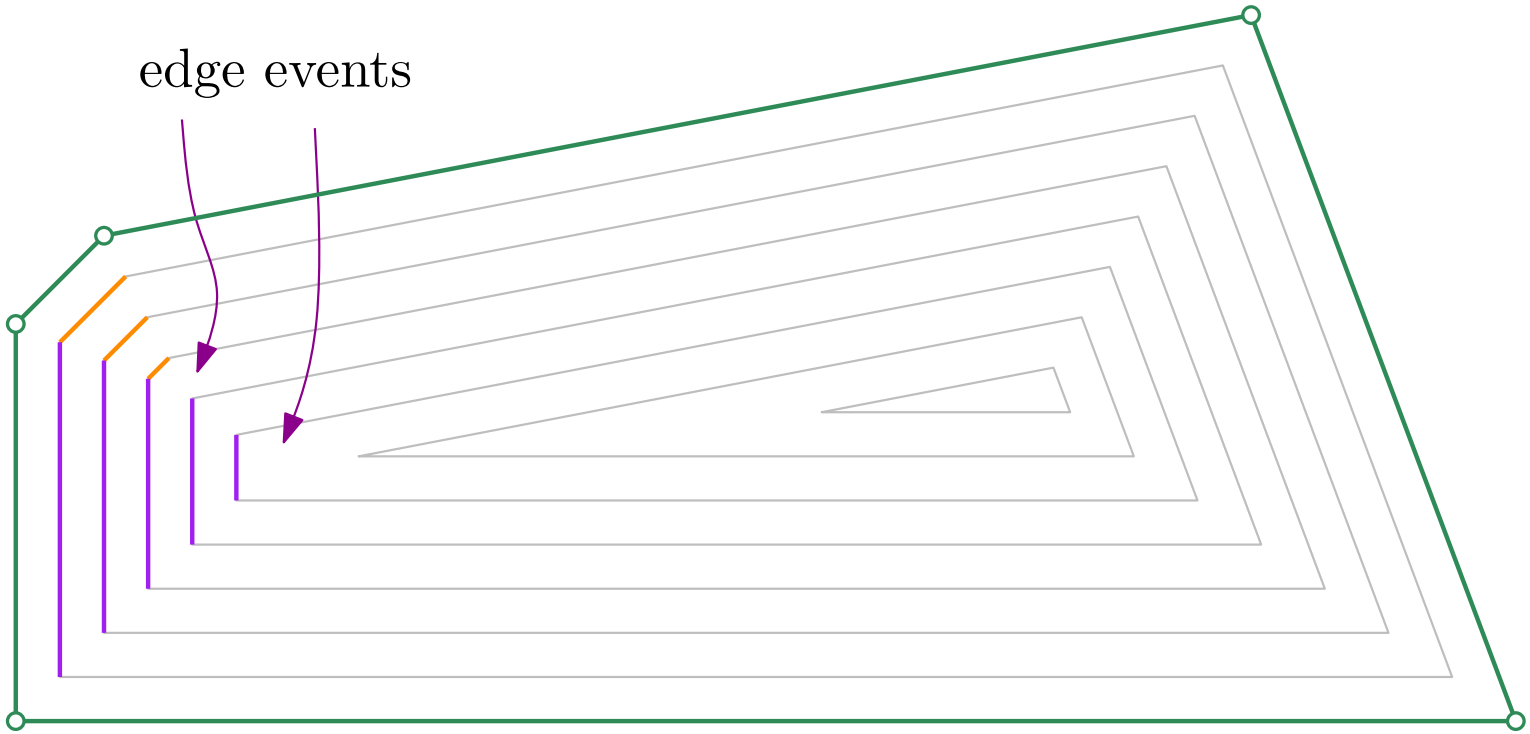
Edge event

- Wavefront topology changes over time.
- *Edge event*: an edge of $\mathcal{WF}(\mathcal{P}, t)$ vanishes.



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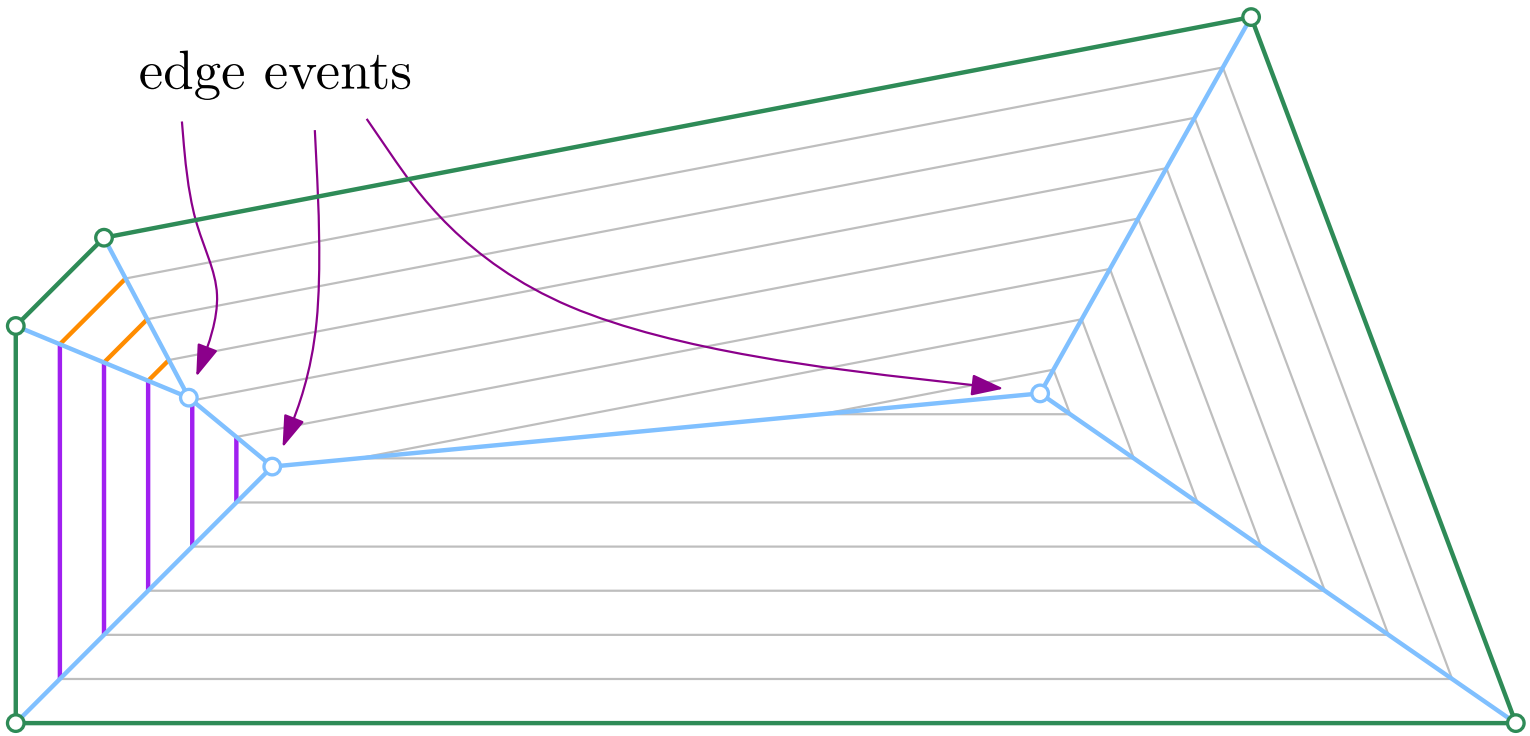
Change of Wavefront Topology

Edge event

- Wavefront topology changes over time.
- *Edge event*: an edge of $\mathcal{WF}(\mathcal{P}, t)$ vanishes.
- Such a change of topology corresponds to a *node* of $\mathcal{SK}(\mathcal{P})$.



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Change of Wavefront Topology

Split event

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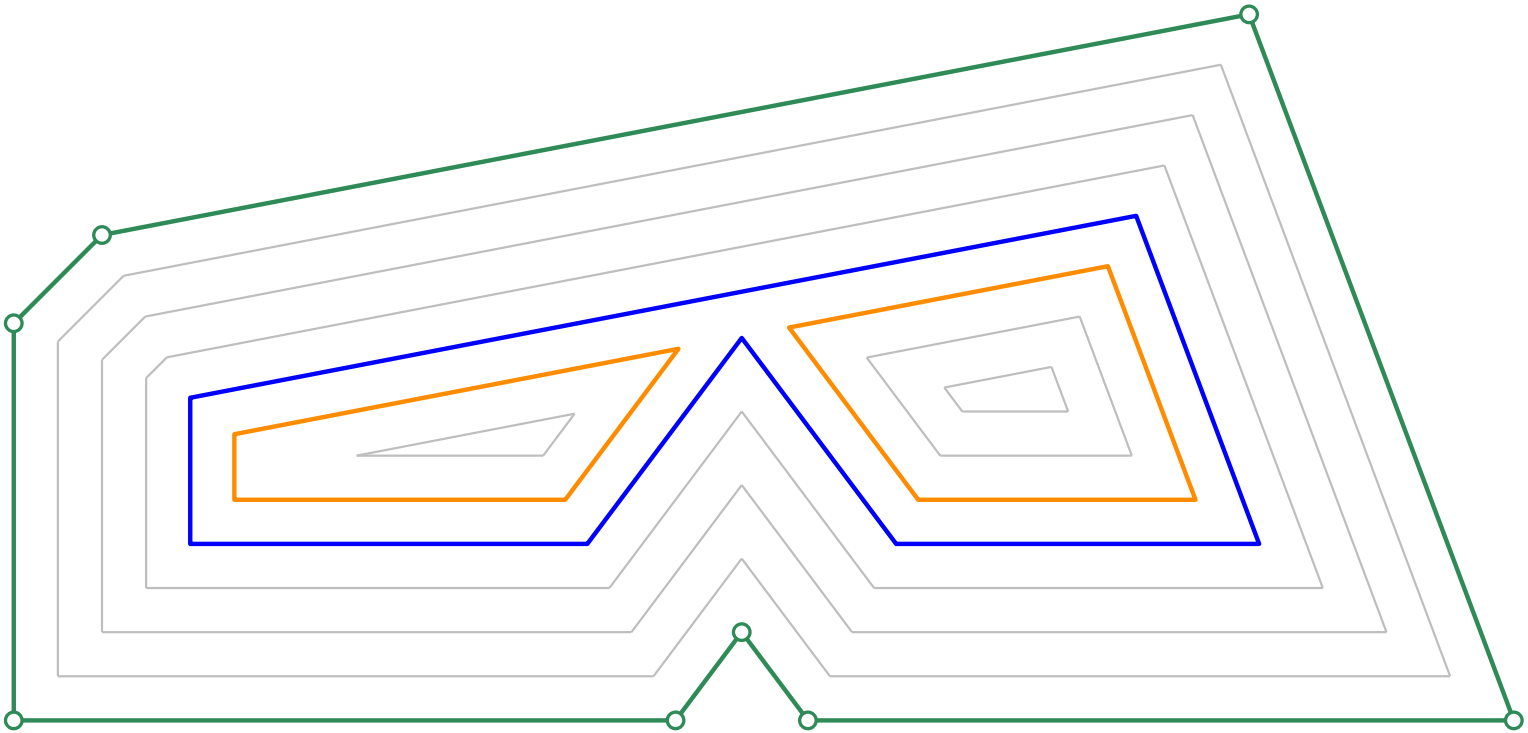

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- Wavefront topology changes over time.
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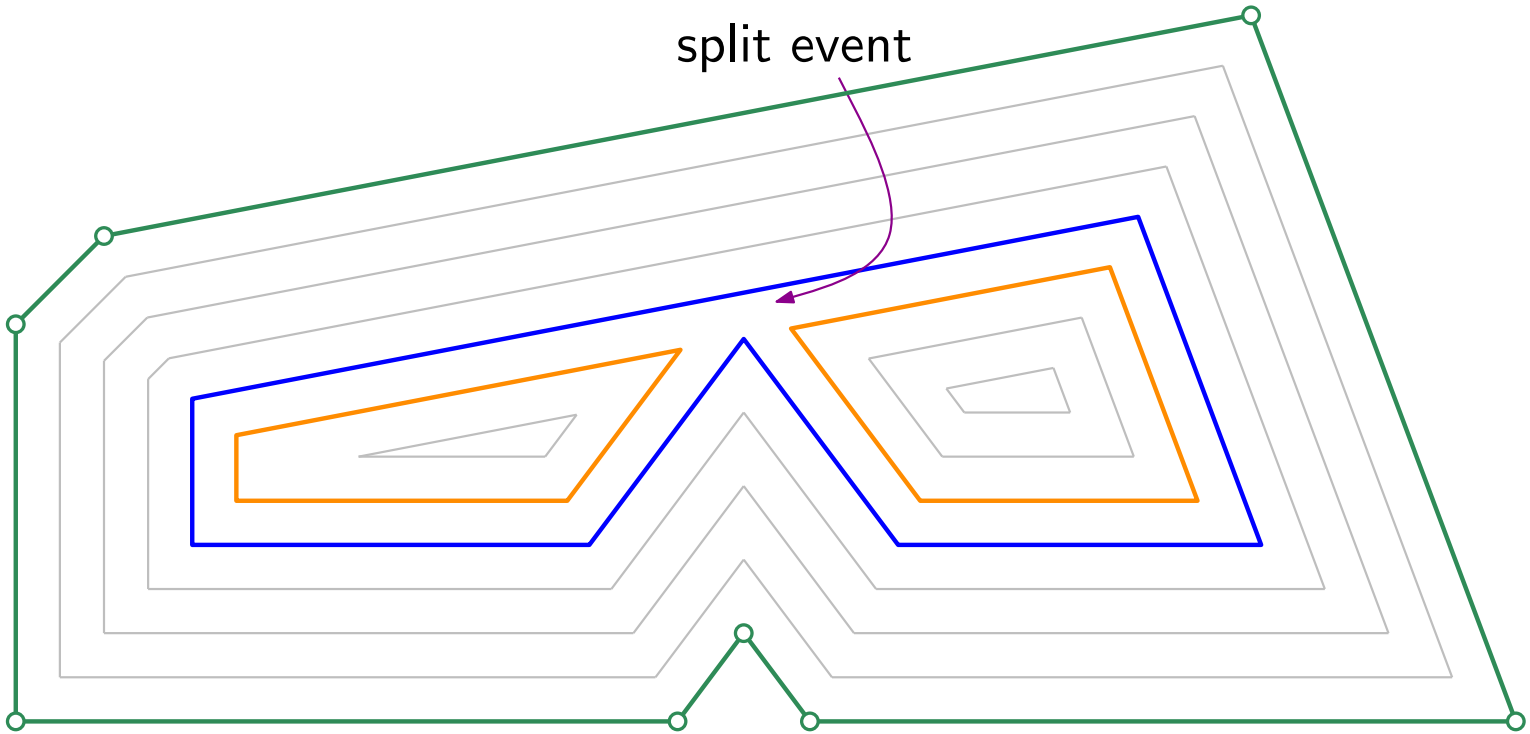

Change of Wavefront Topology

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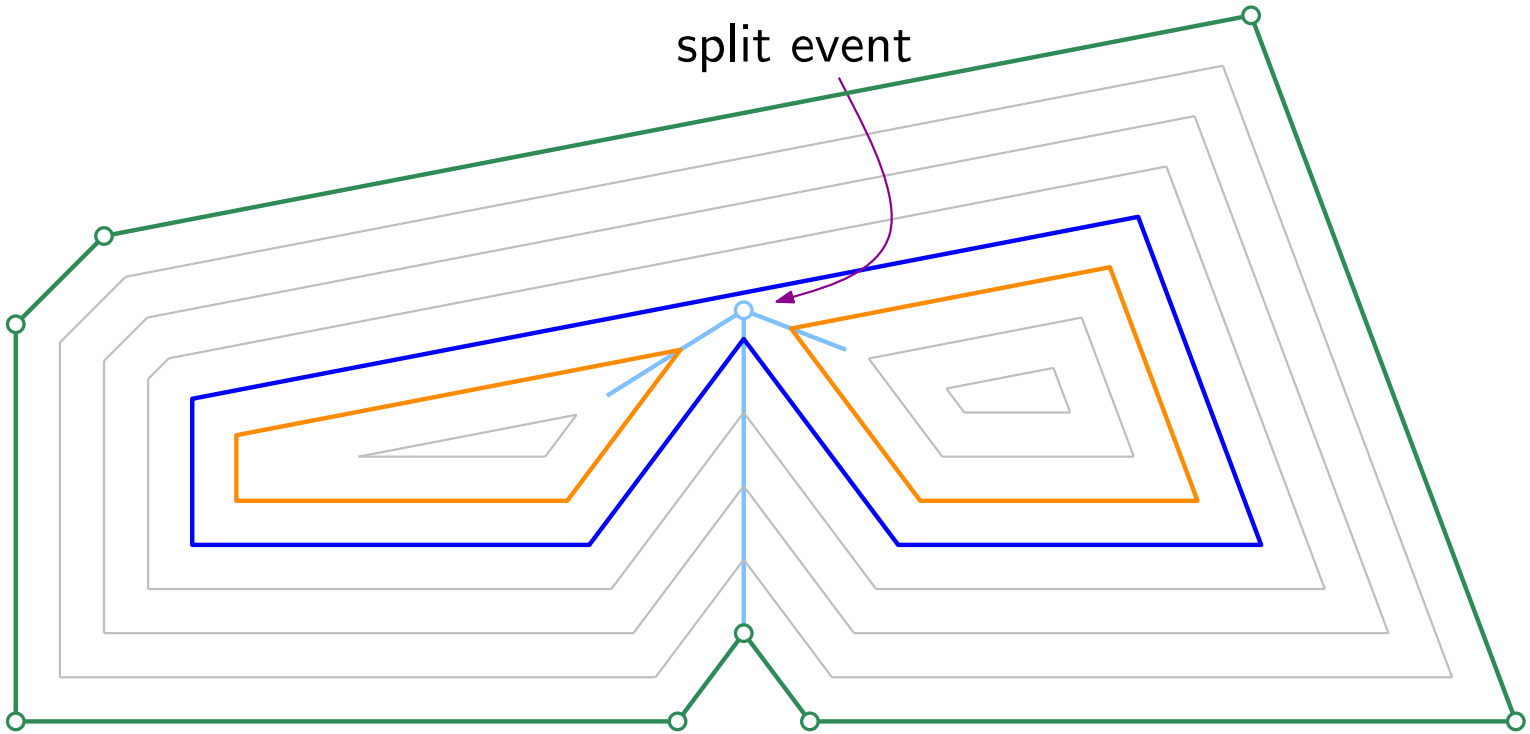
- Wavefront topology changes over time.
- *Split event*: wavefront splits into two parts.
- Also split events correspond to *nodes* of $\mathcal{SK}(\mathcal{P})$.



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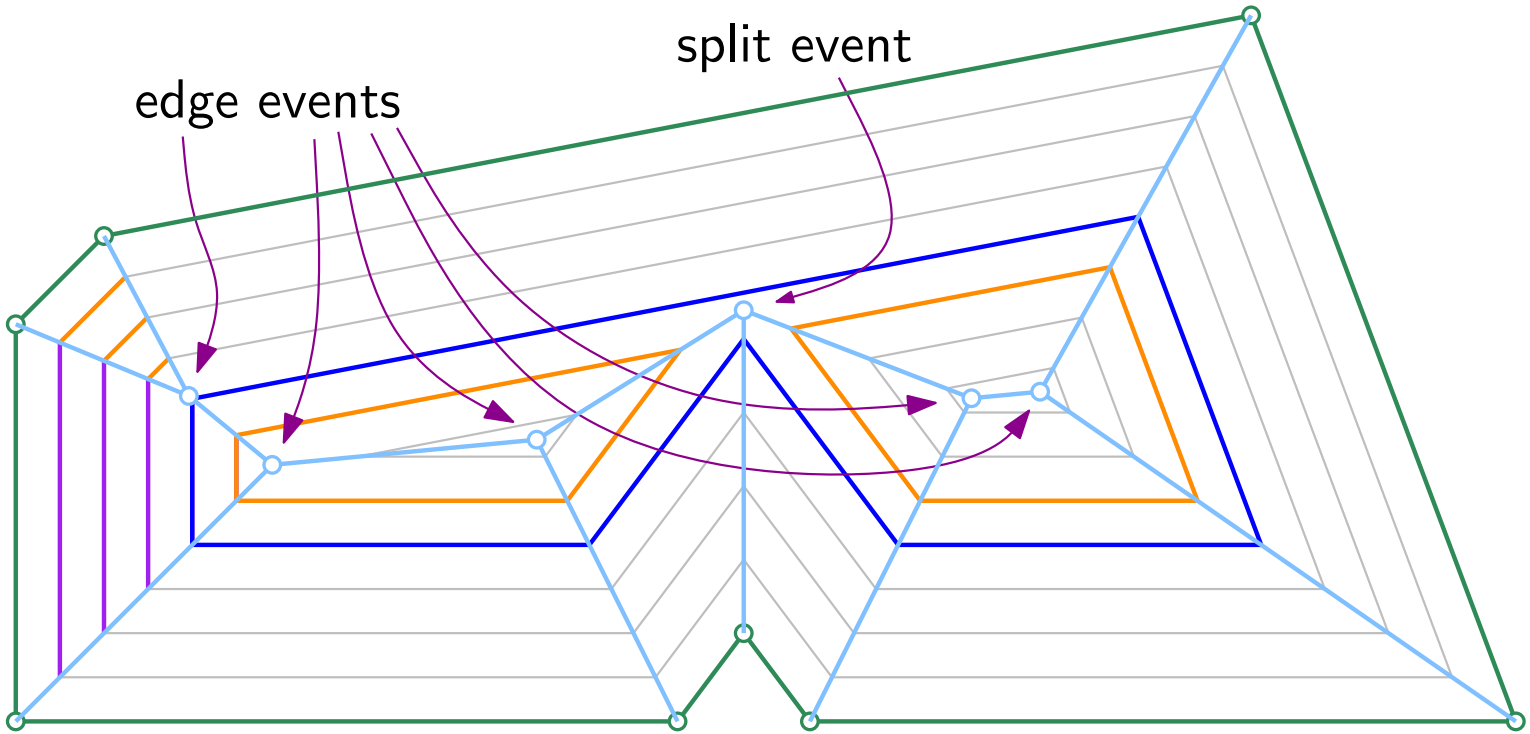

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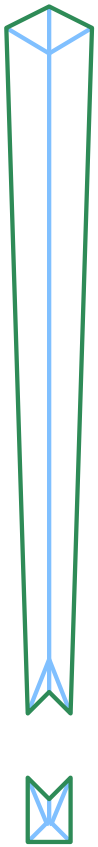
Straight Skeleton



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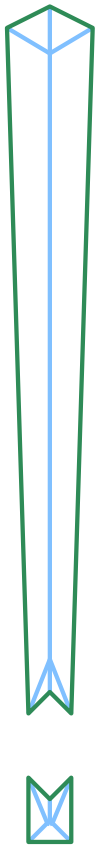

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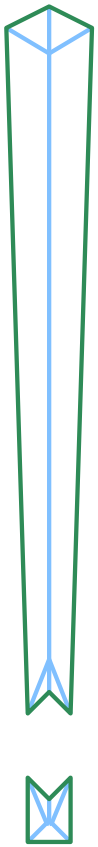

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Basic facts

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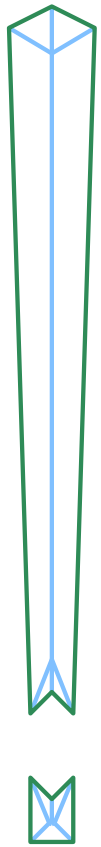

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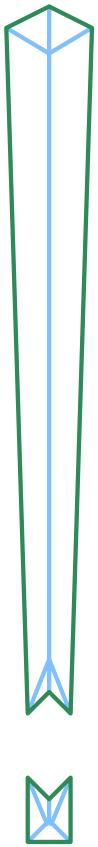

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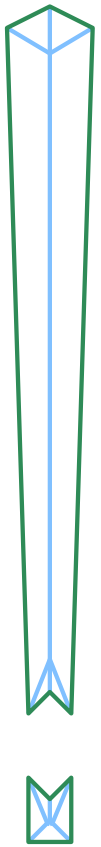

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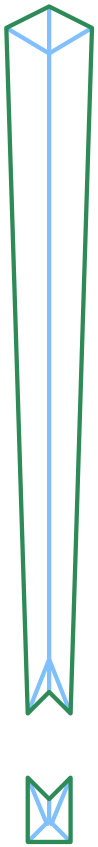

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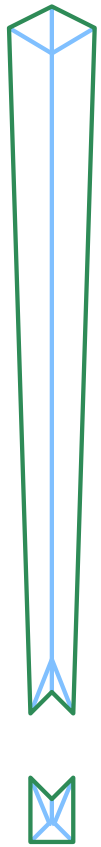
- These events correspond to nodes of $\mathcal{SK}(\mathcal{P})$.
- No metric-based definition of straight skeletons exists.



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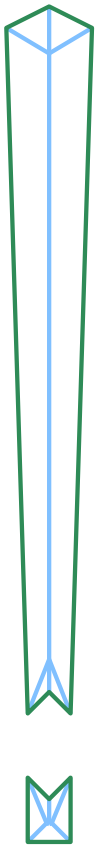

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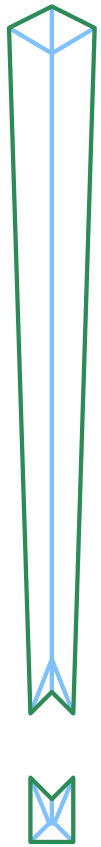
- If $\mathcal{P}$ has n segments then $\mathcal{SK}(\mathcal{P})$ consists of $O(n)$ nodes and $O(n)$ straight-line edges.



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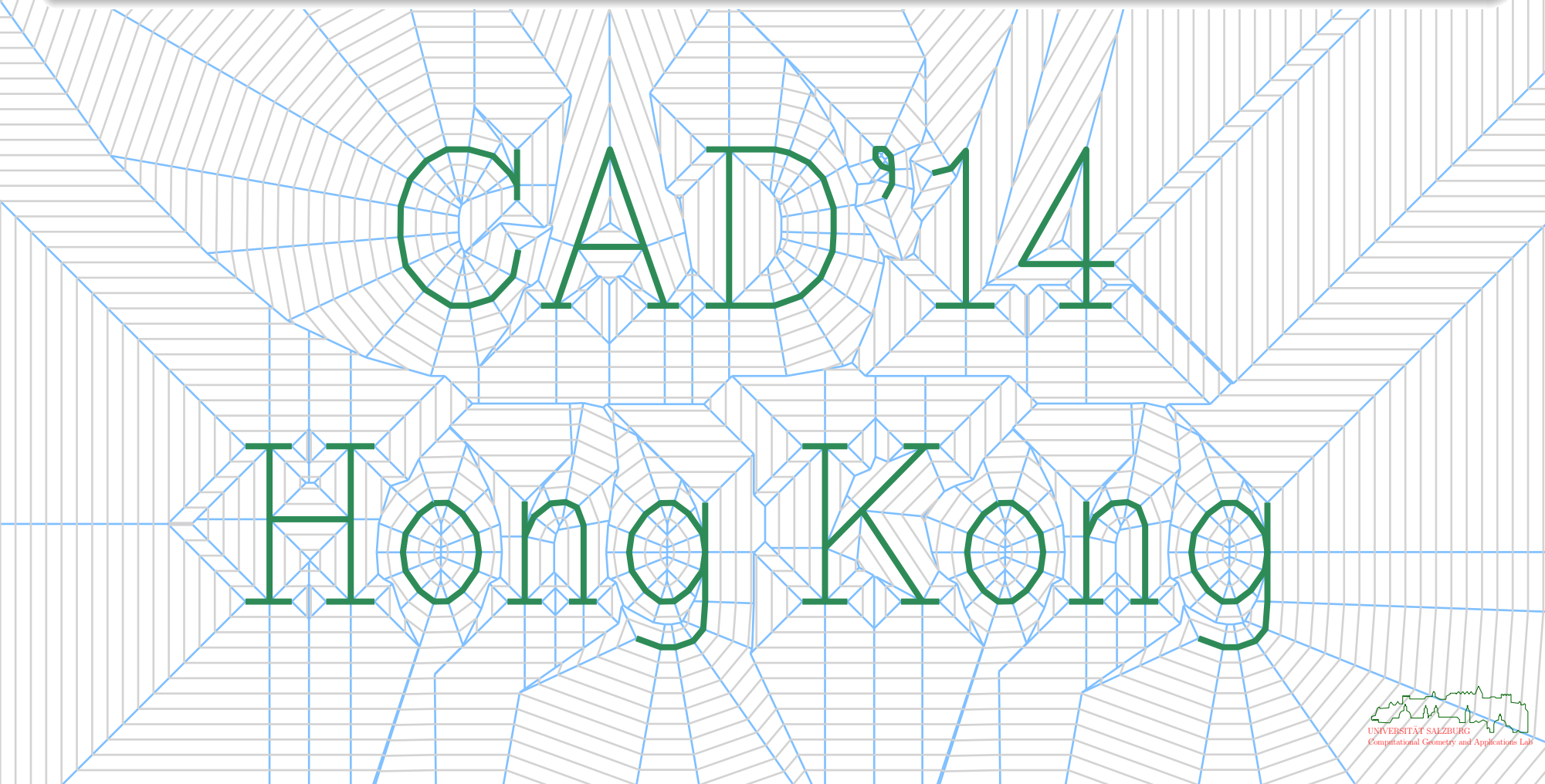
Straight Skeleton of a PSLG

The definition of straight skeletons can be extended easily to arbitrary planar straight line graphs (PSLGs) within the entire plane, i.e., to a collection of straight-line segments that do not intersect except possibly at common endpoints.



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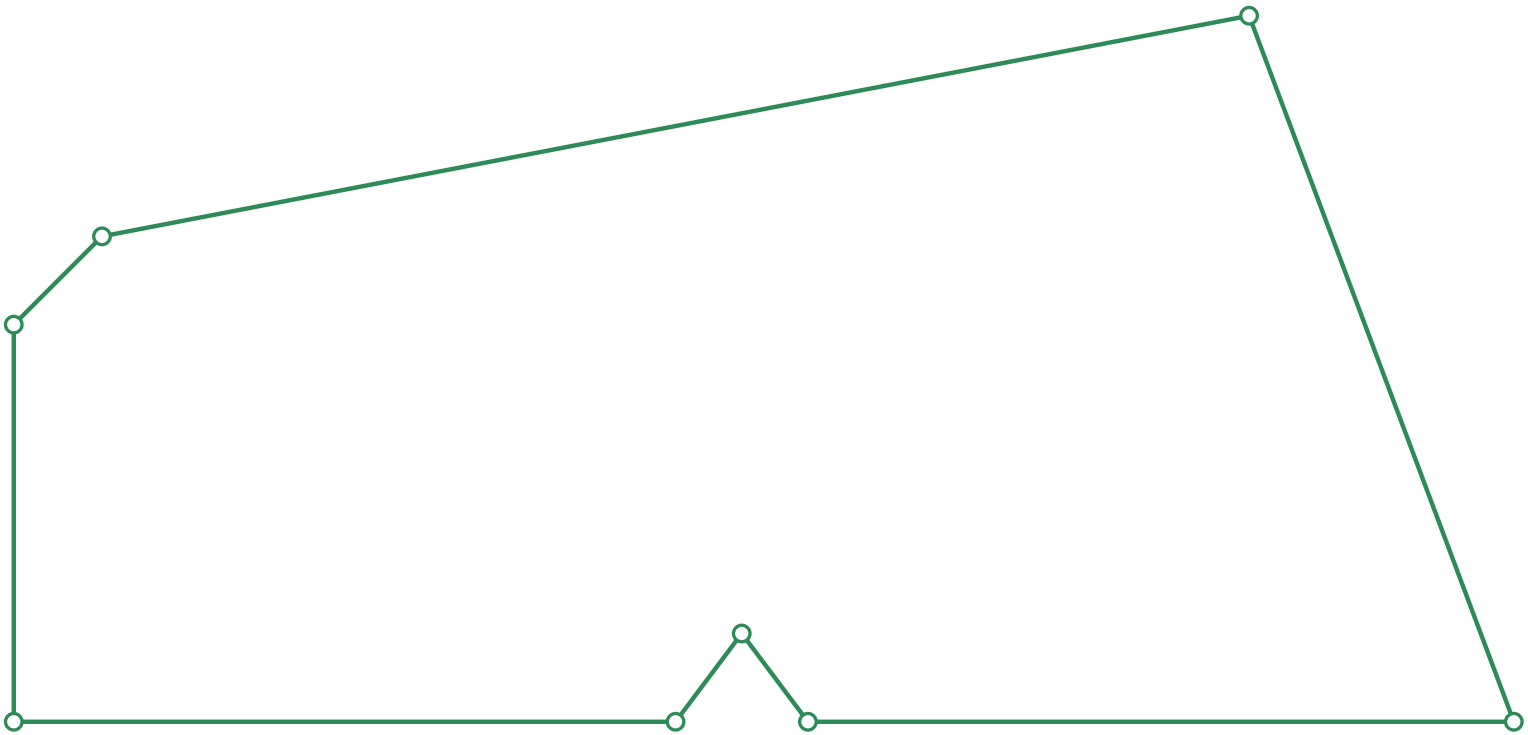

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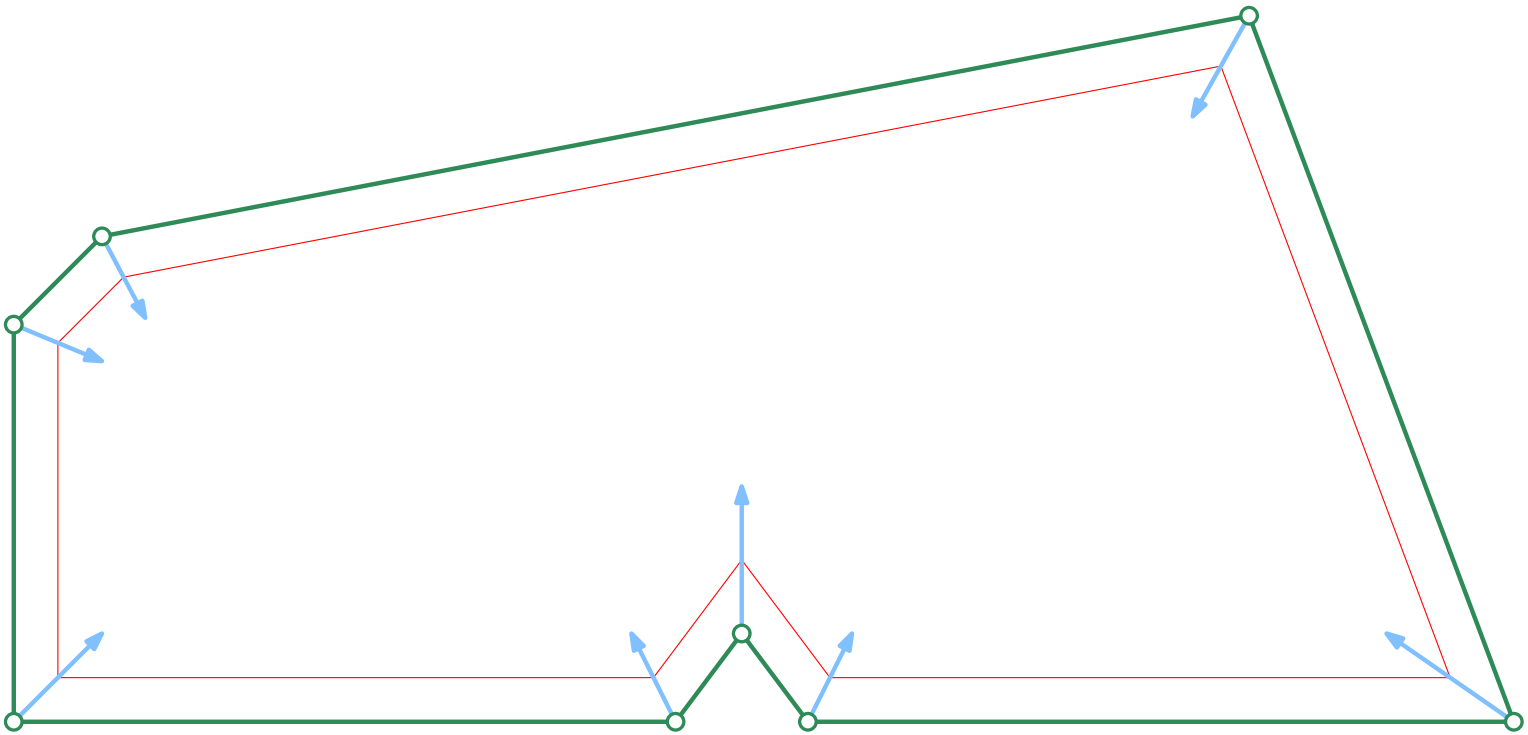
Computing Straight Skeletons

Basic idea

- Simulate the wavefront propagation.



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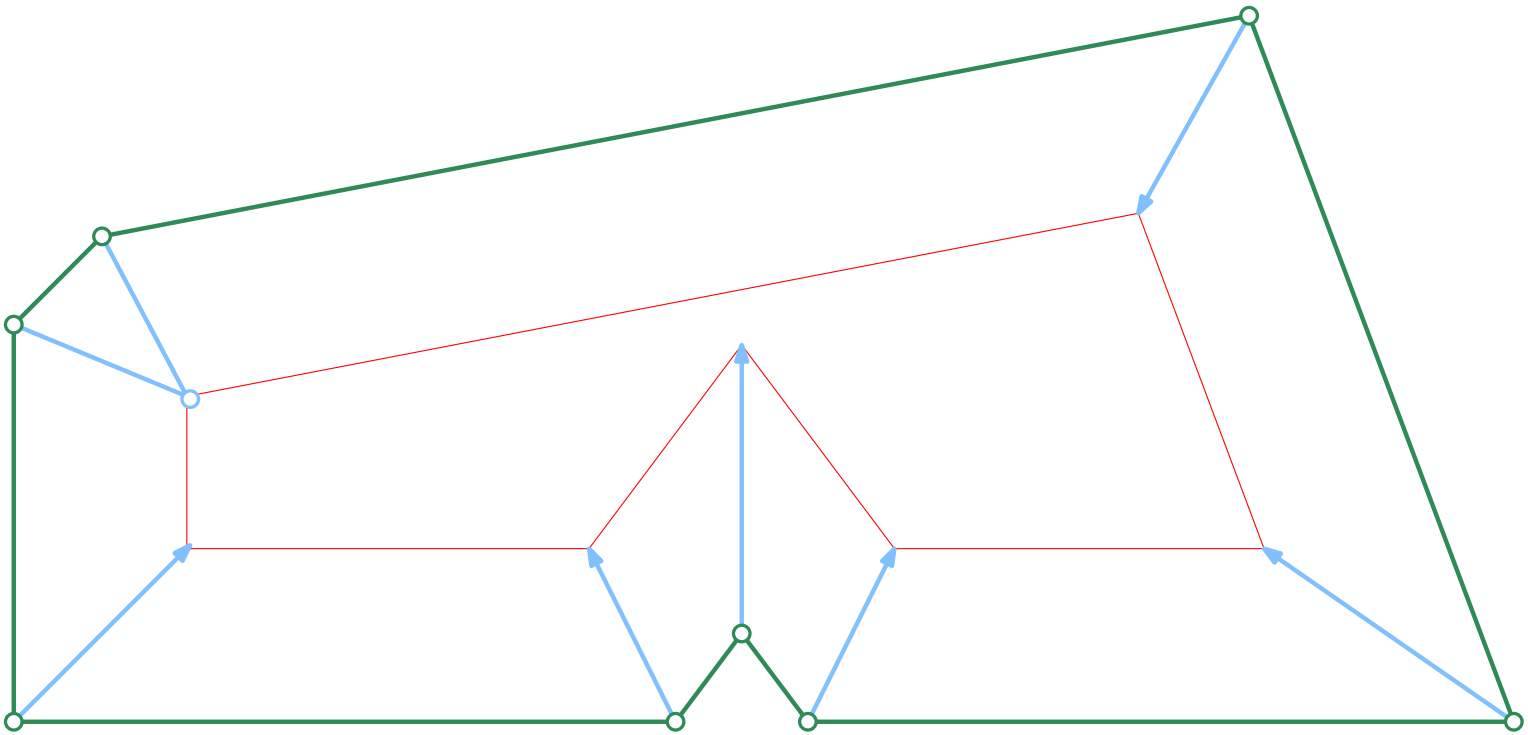
Computing Straight Skeletons

Basic idea

- Simulate the wavefront propagation.
- Problem: When will the next event happen? Which event?



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Basic idea

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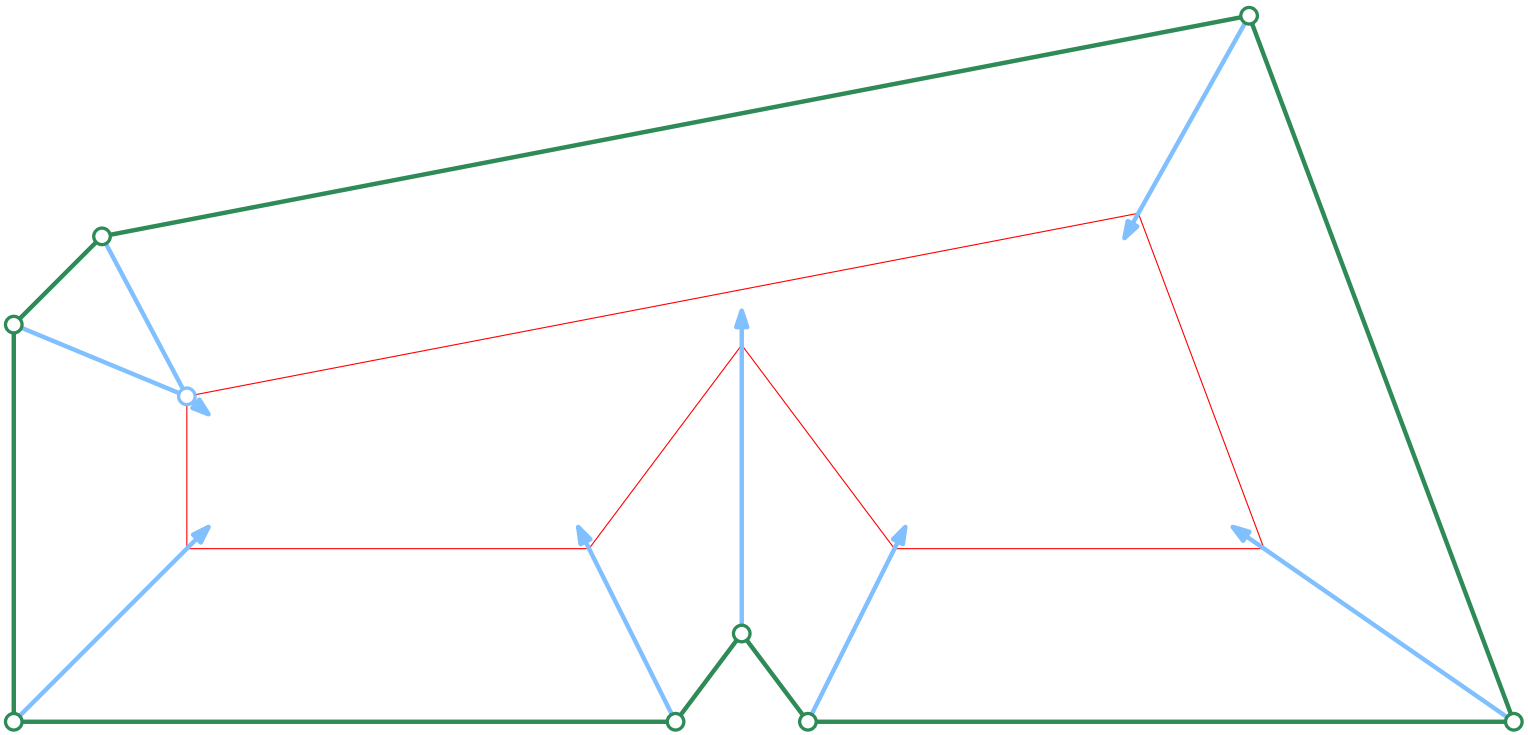
- Problem: When will the next event happen? Which event?
- If we can solve this problem then we can construct straight skeletons.



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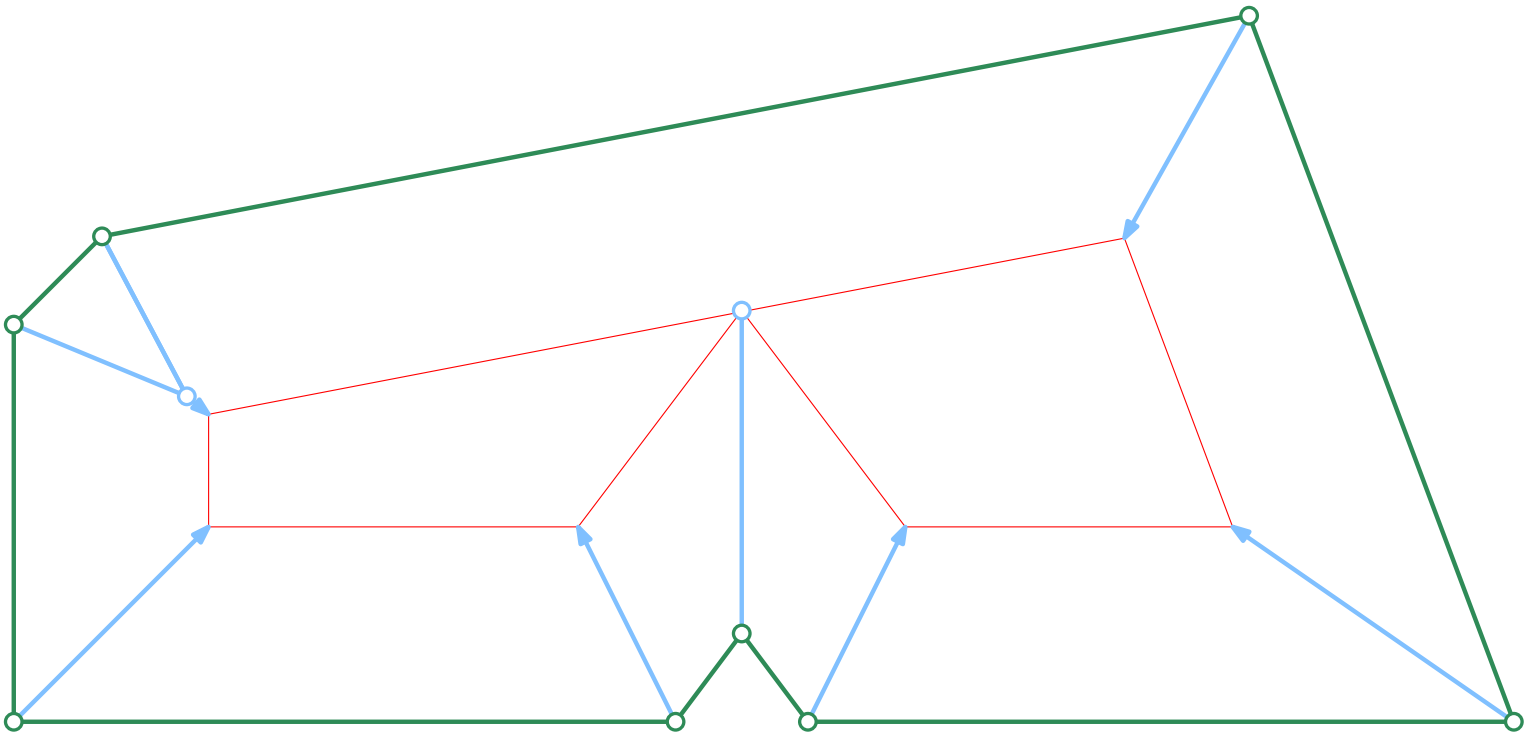

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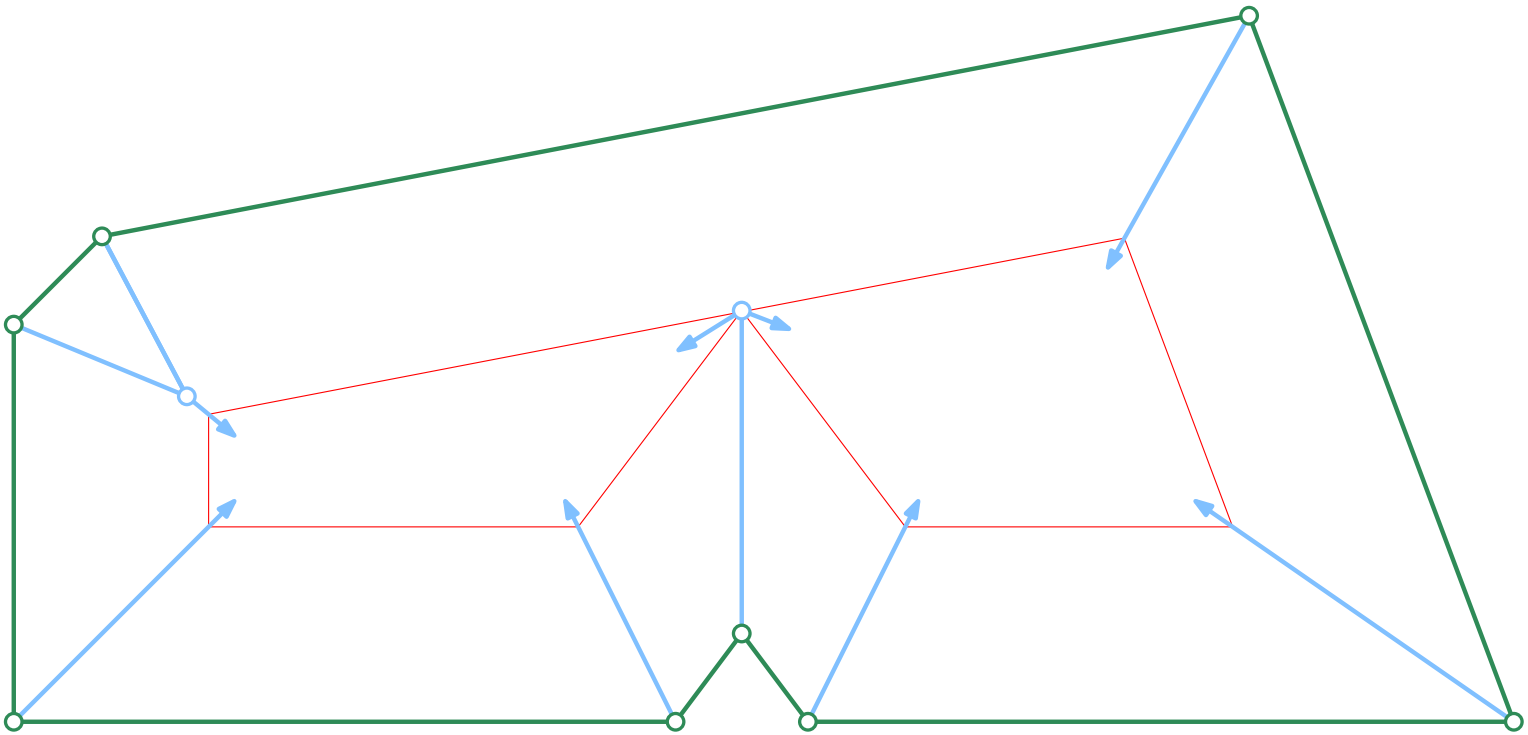

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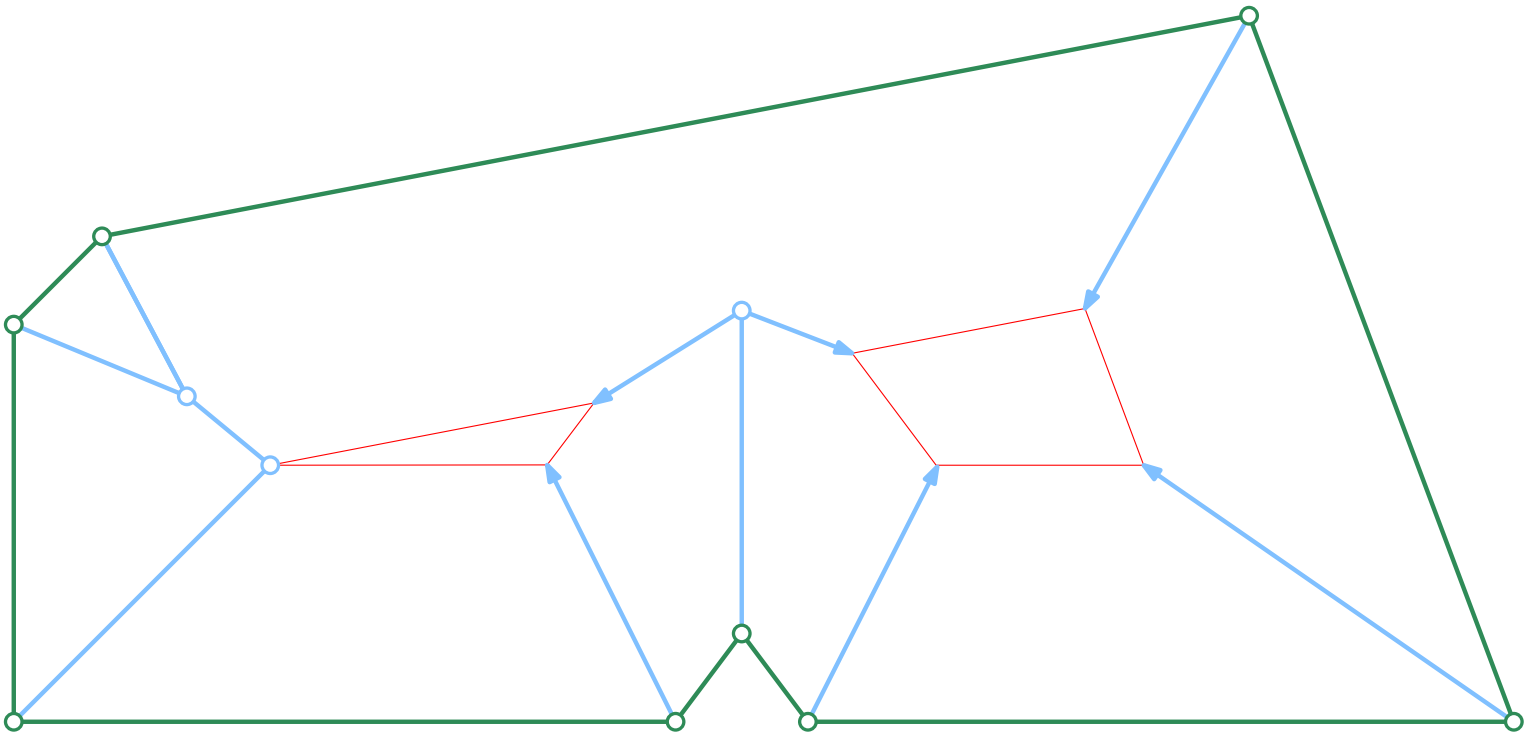

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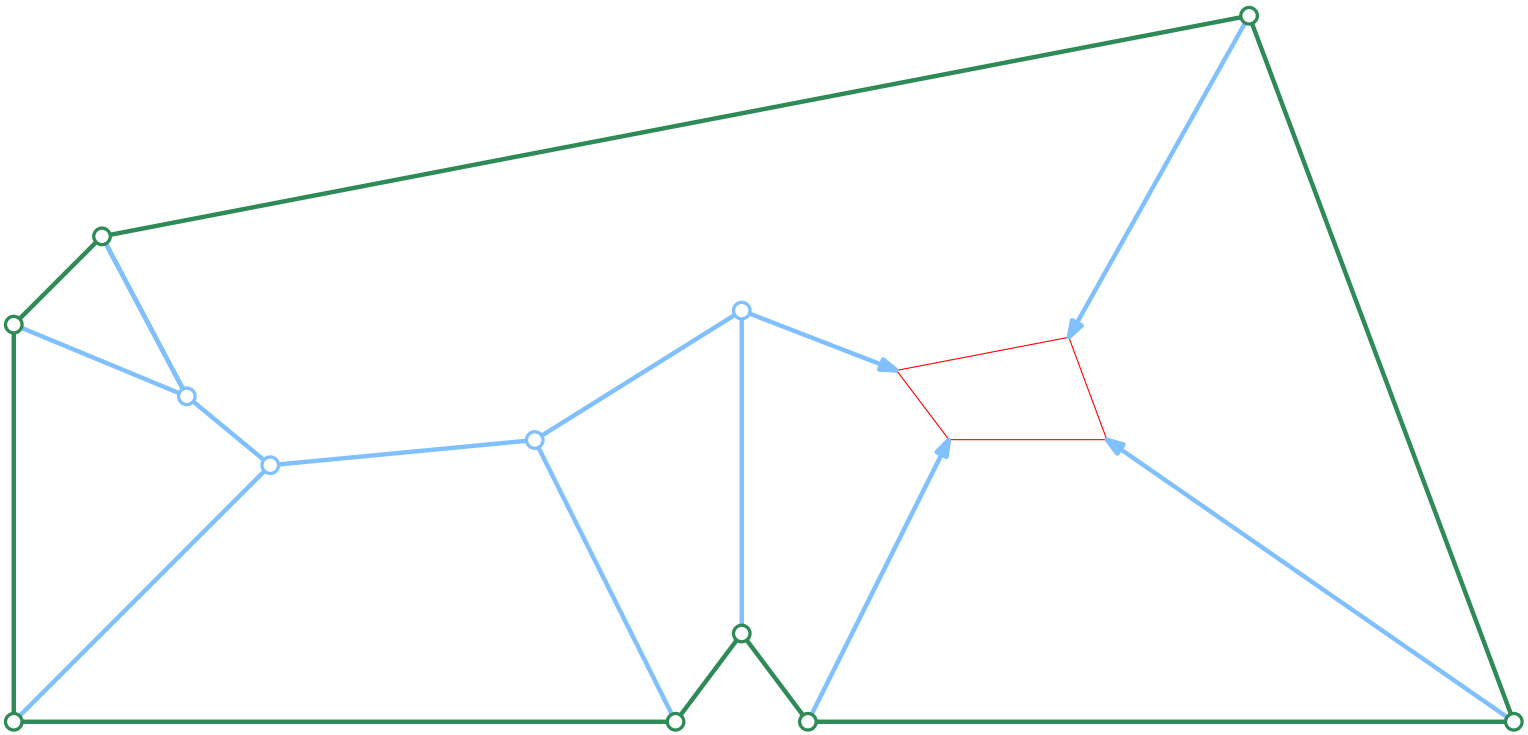

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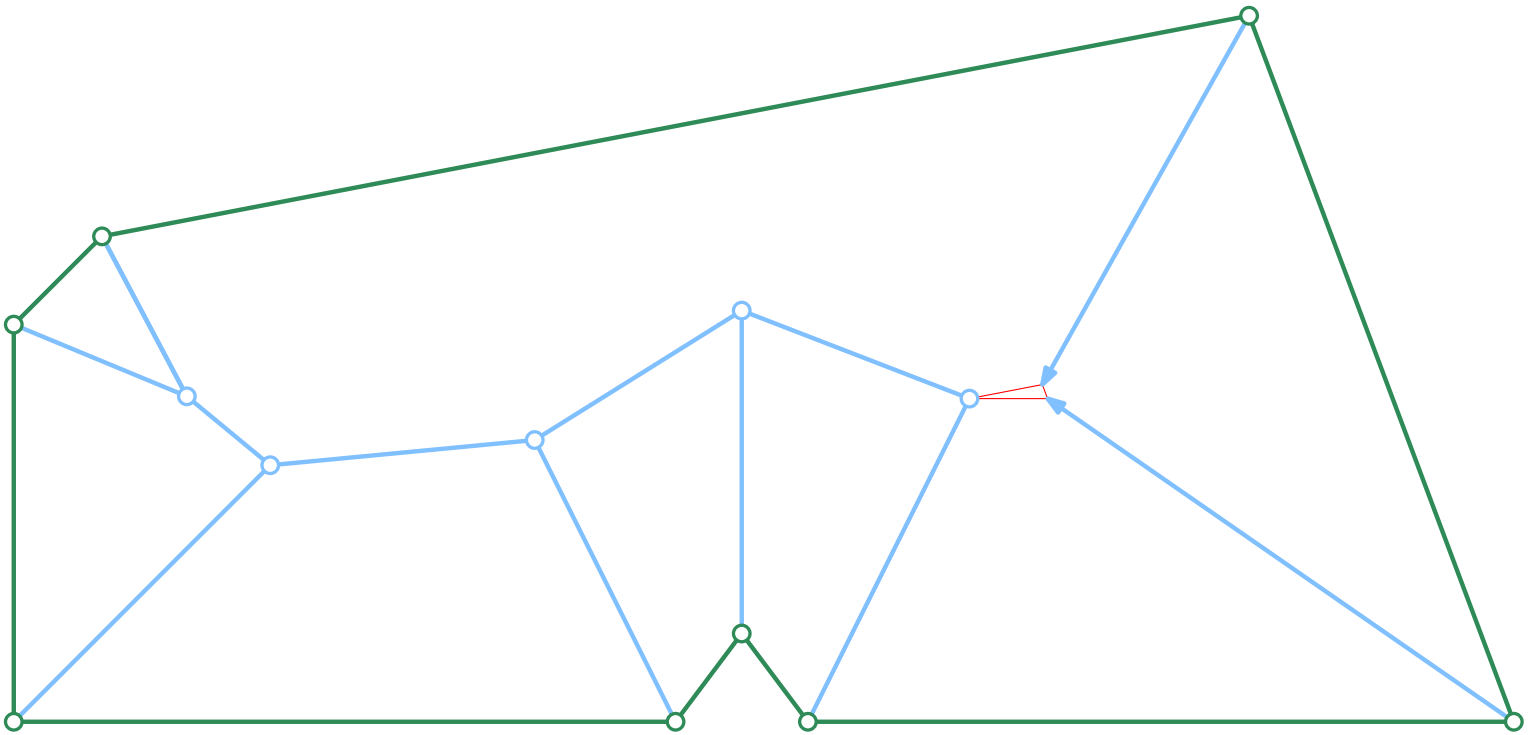

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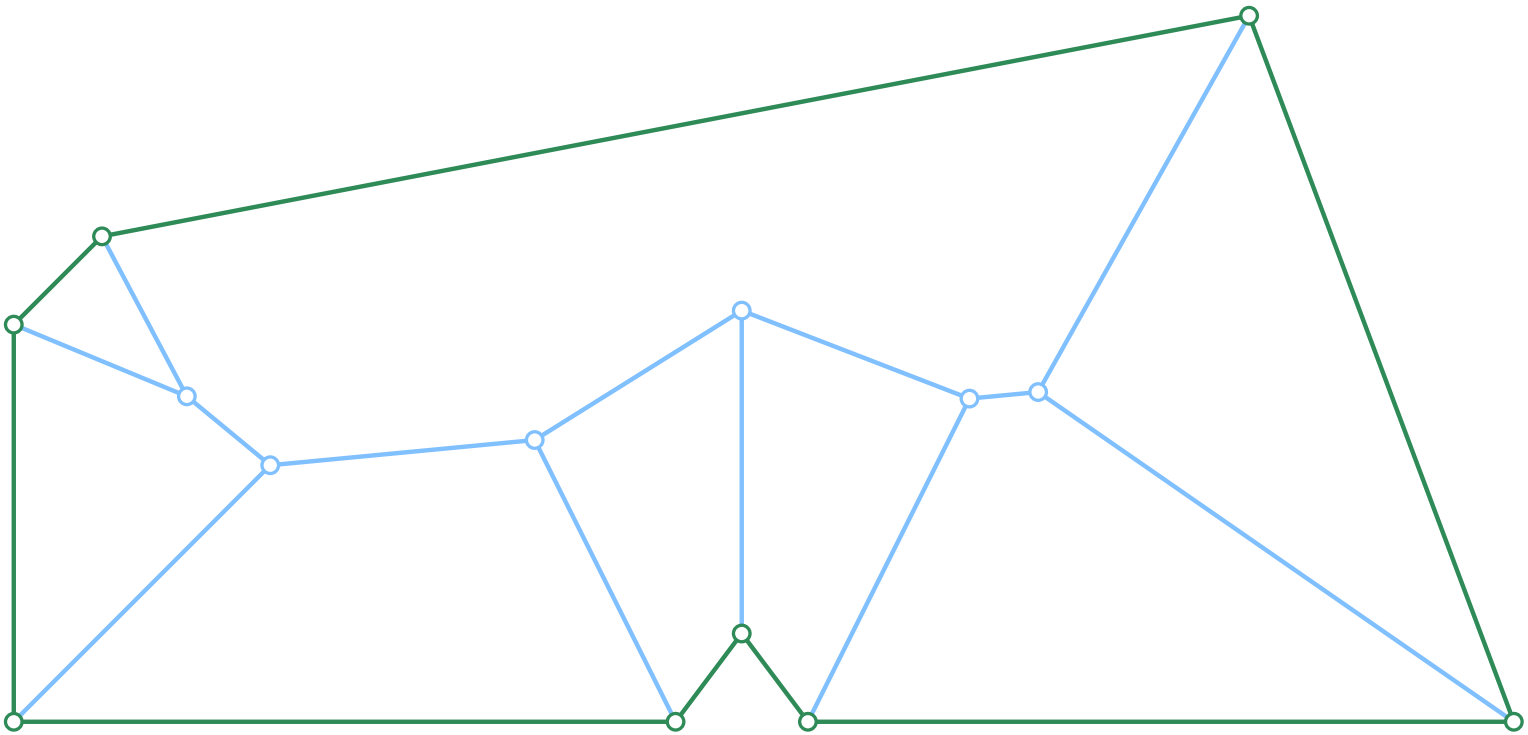
- Problem: When will the next event happen? Which event?
- If we can solve this problem then we can construct straight skeletons.



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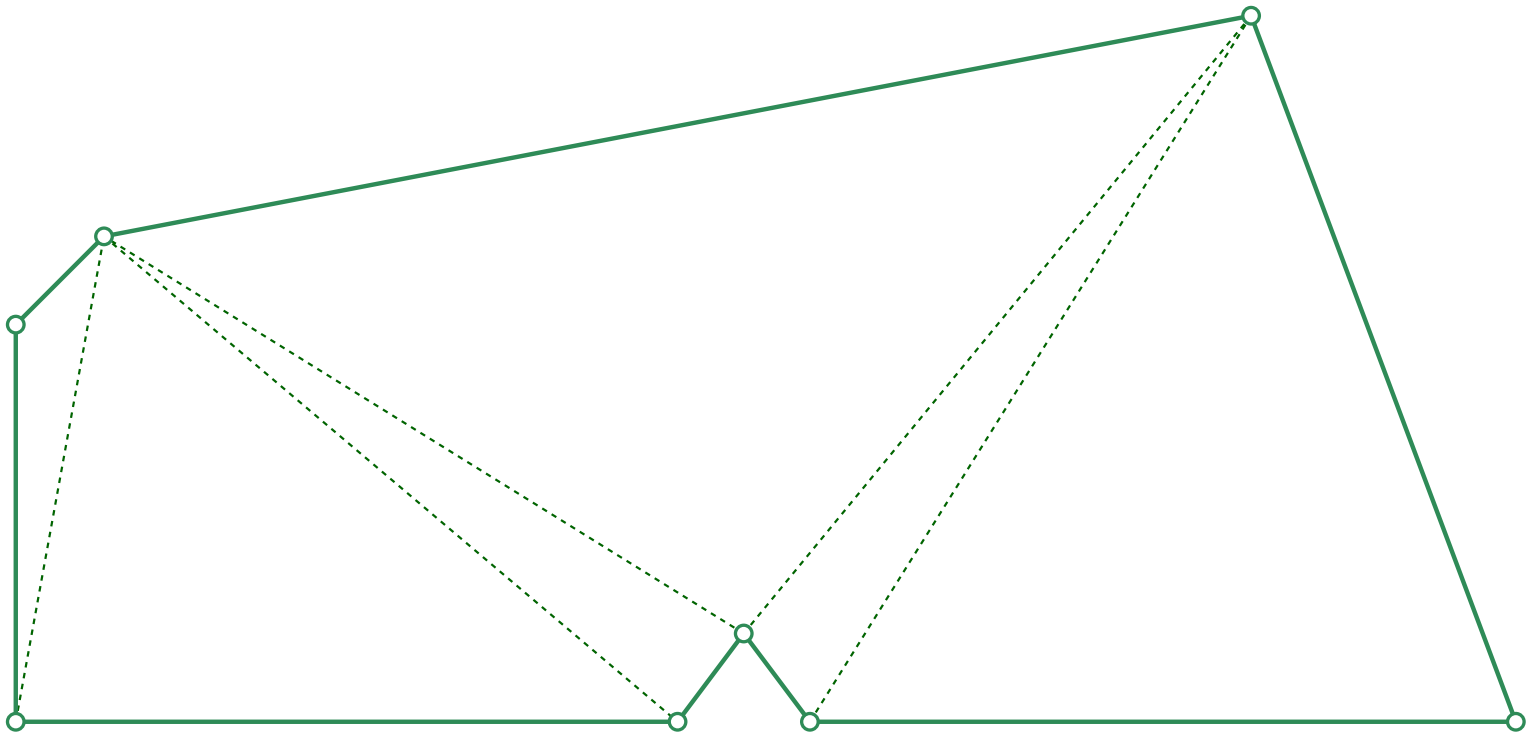
Triangulation-Based Algorithm

Aichholzer&Aurenhammer (1998)

- Maintain a kinetic triangulation of (the interior of) the wavefront.



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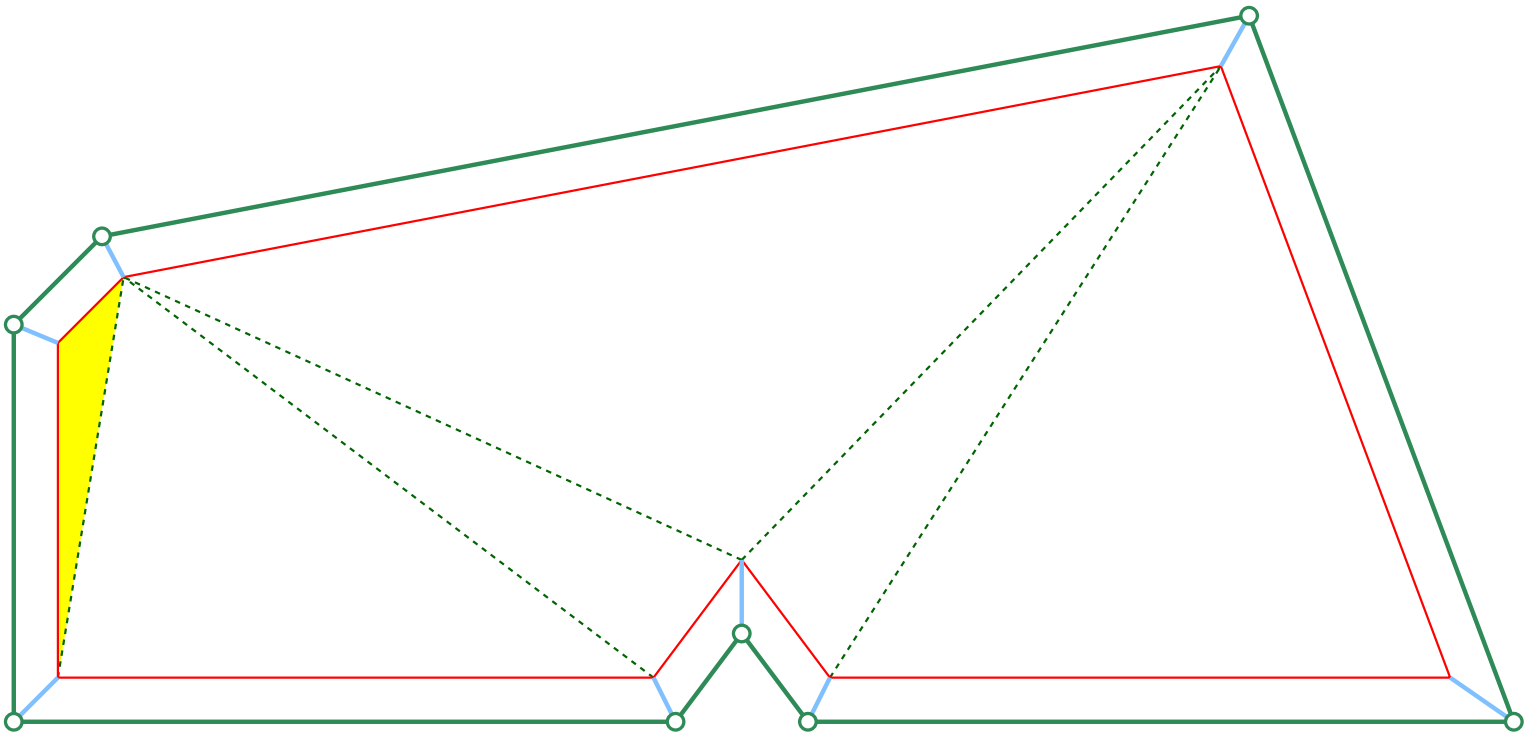

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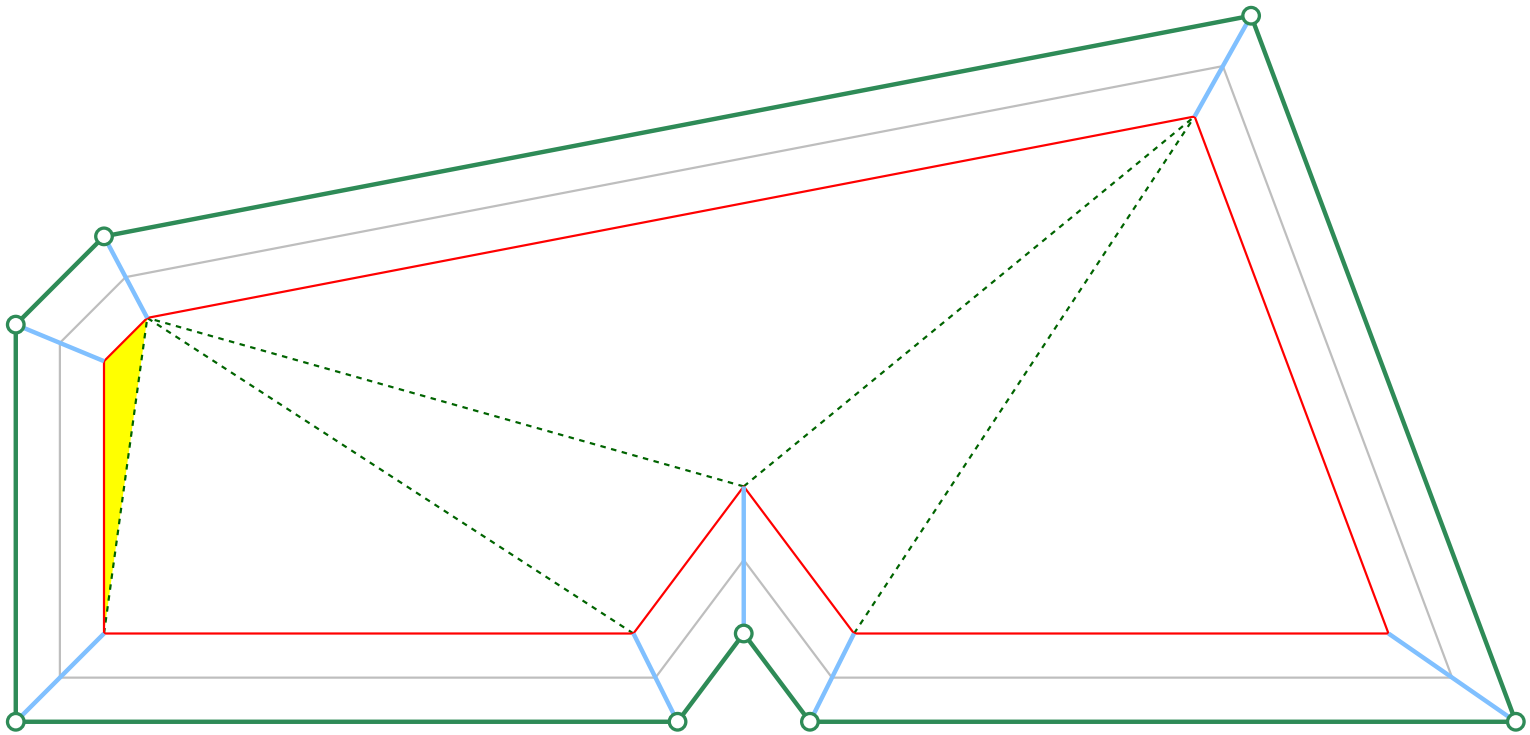

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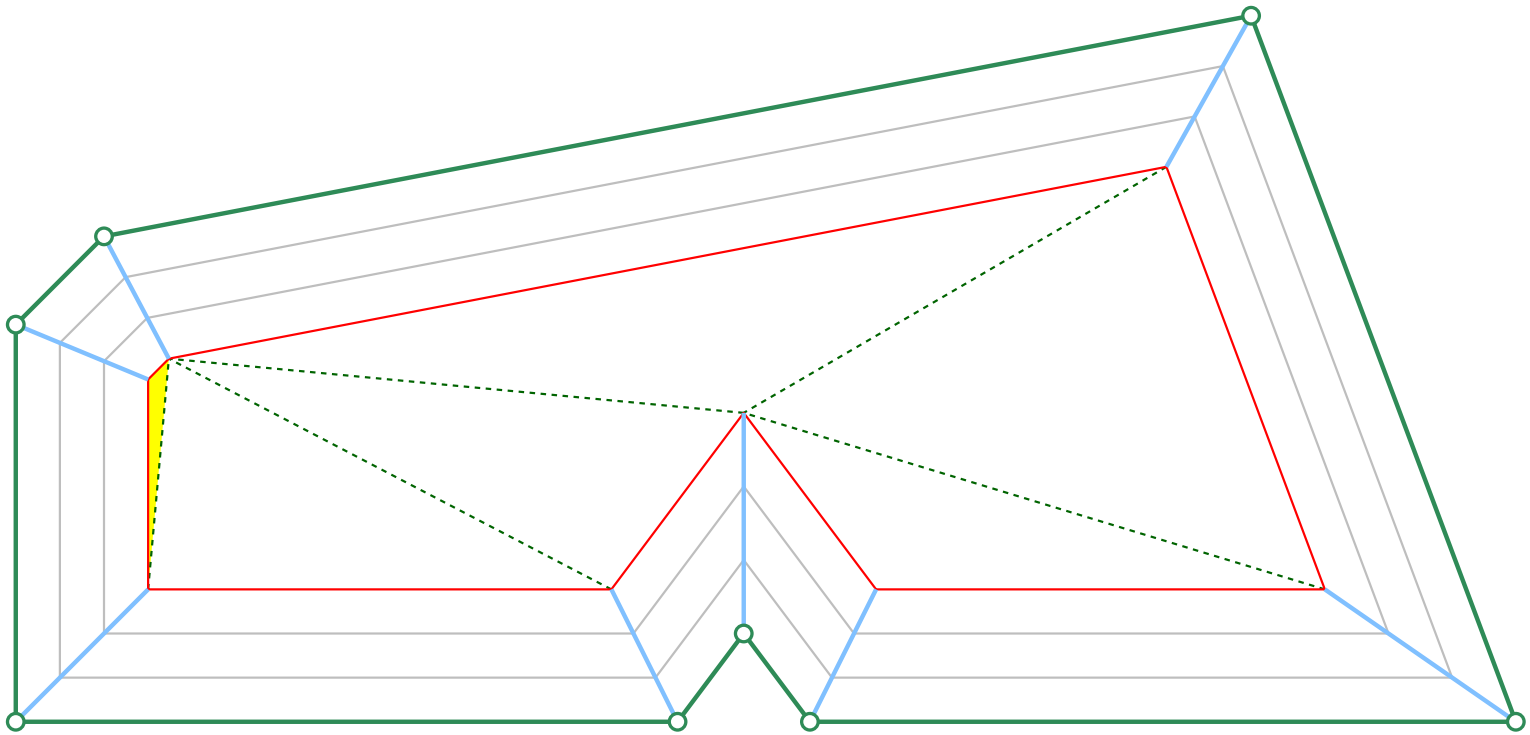

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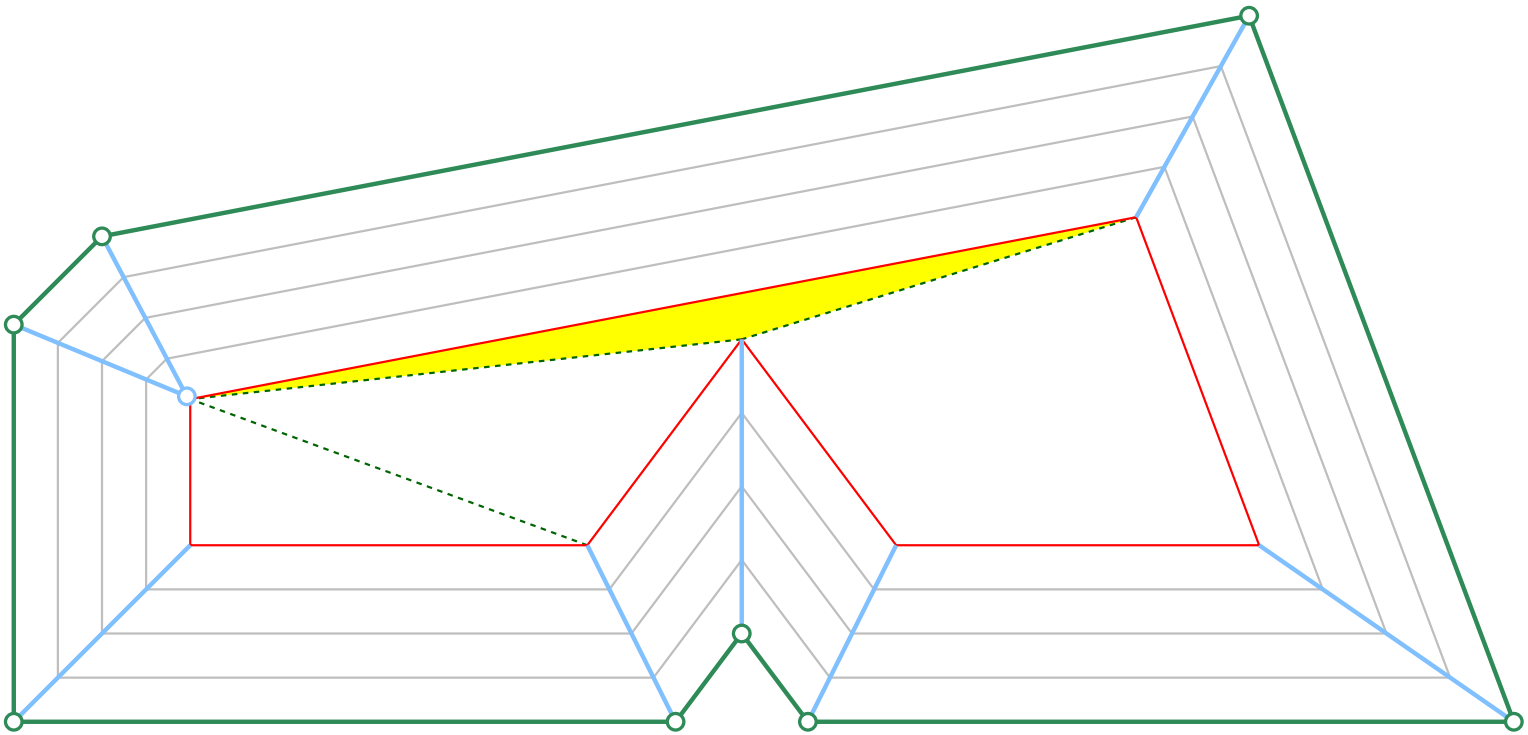

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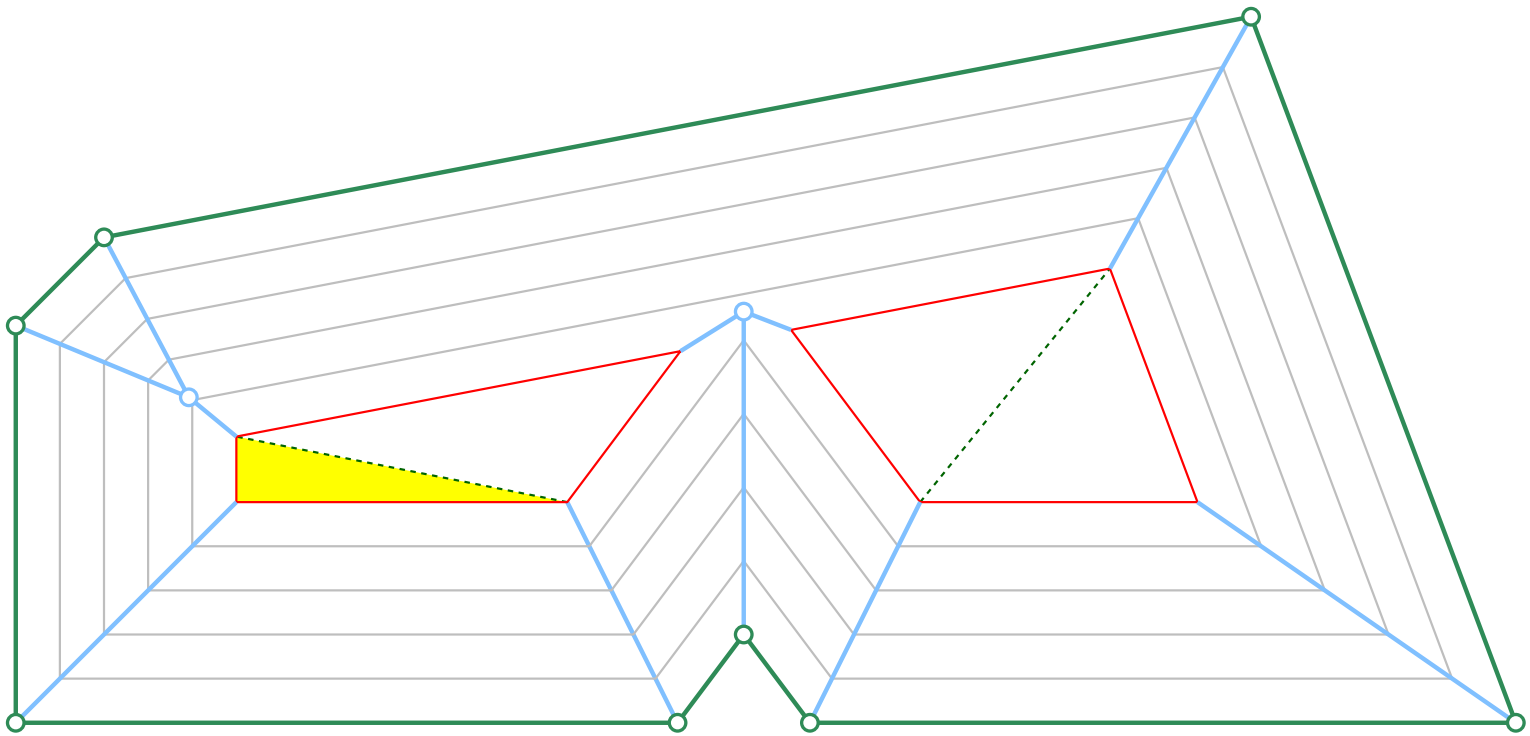

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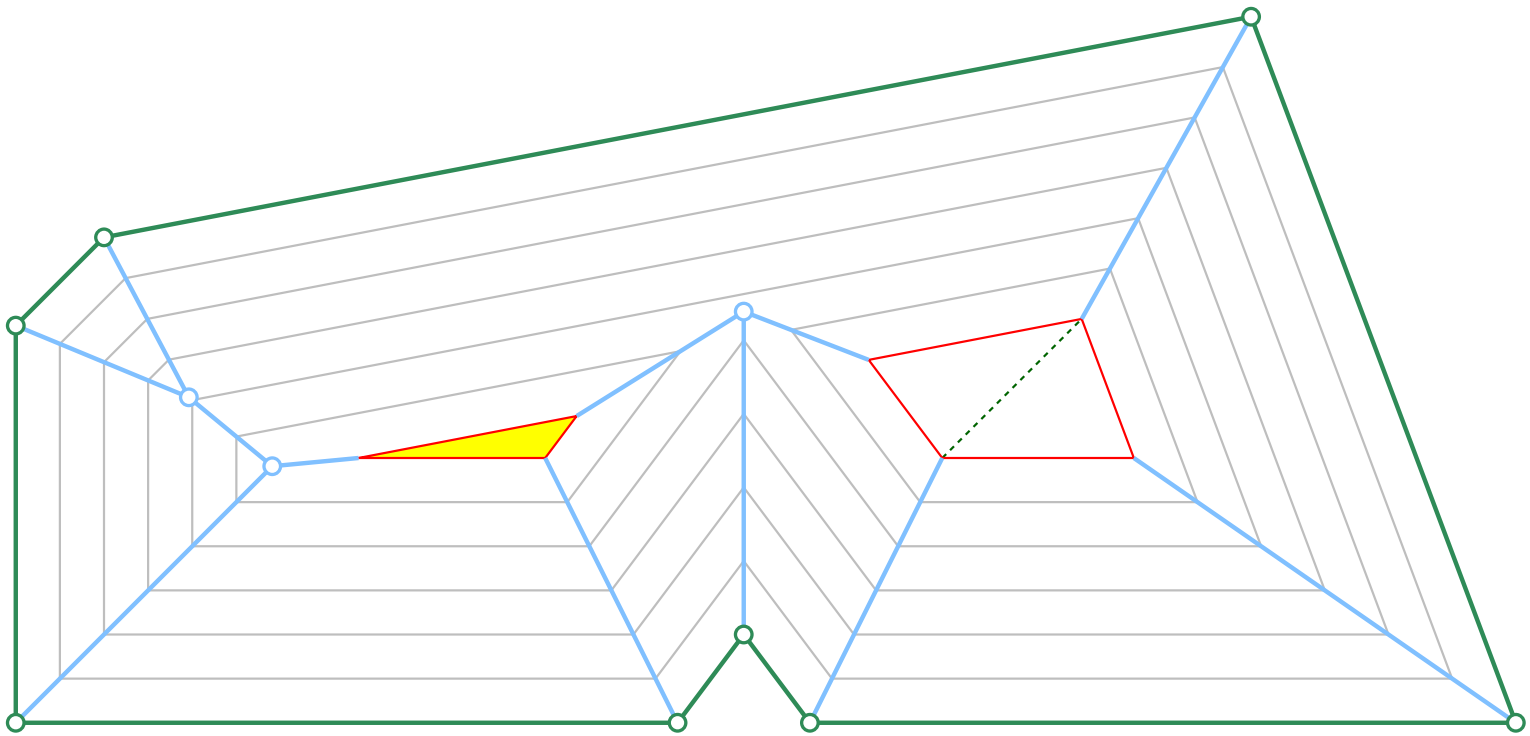

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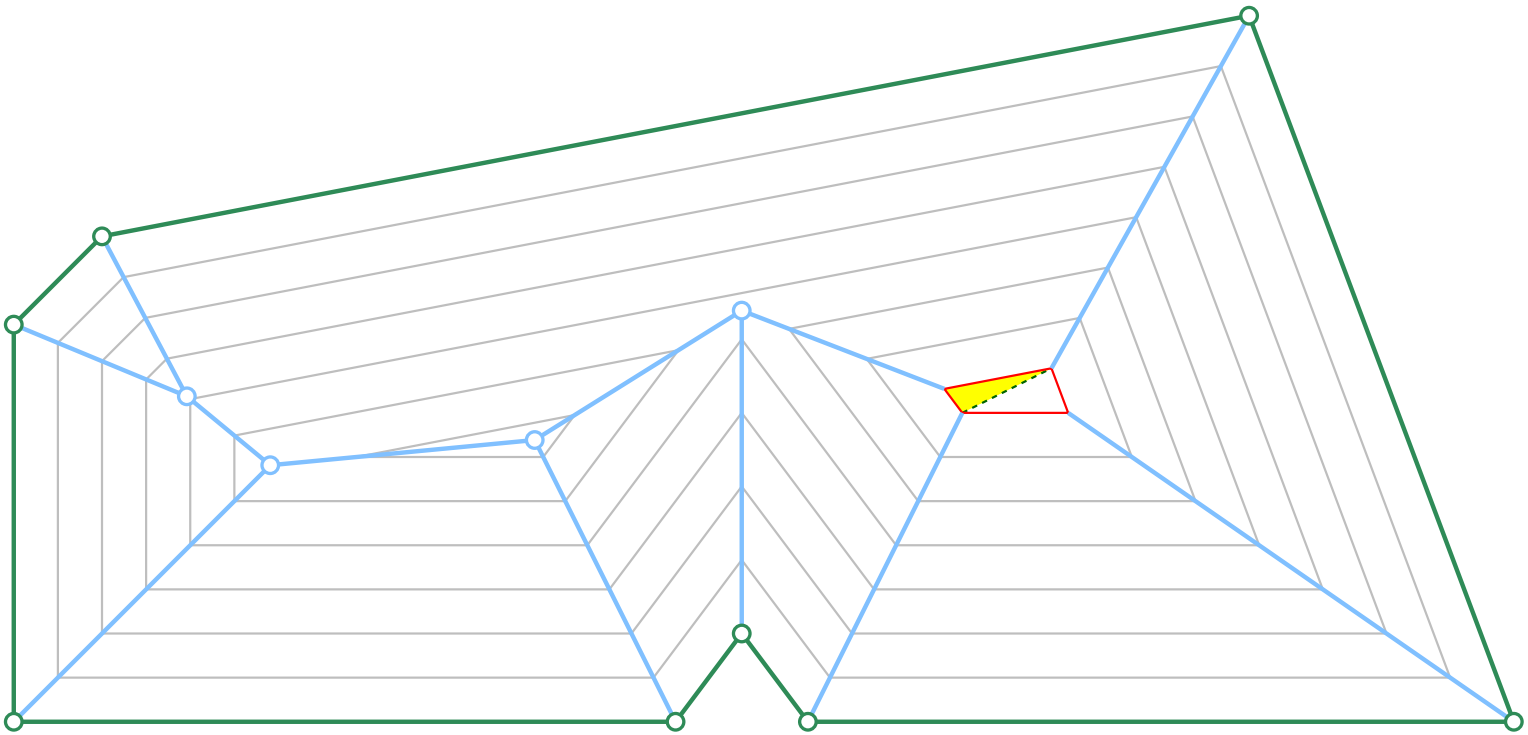

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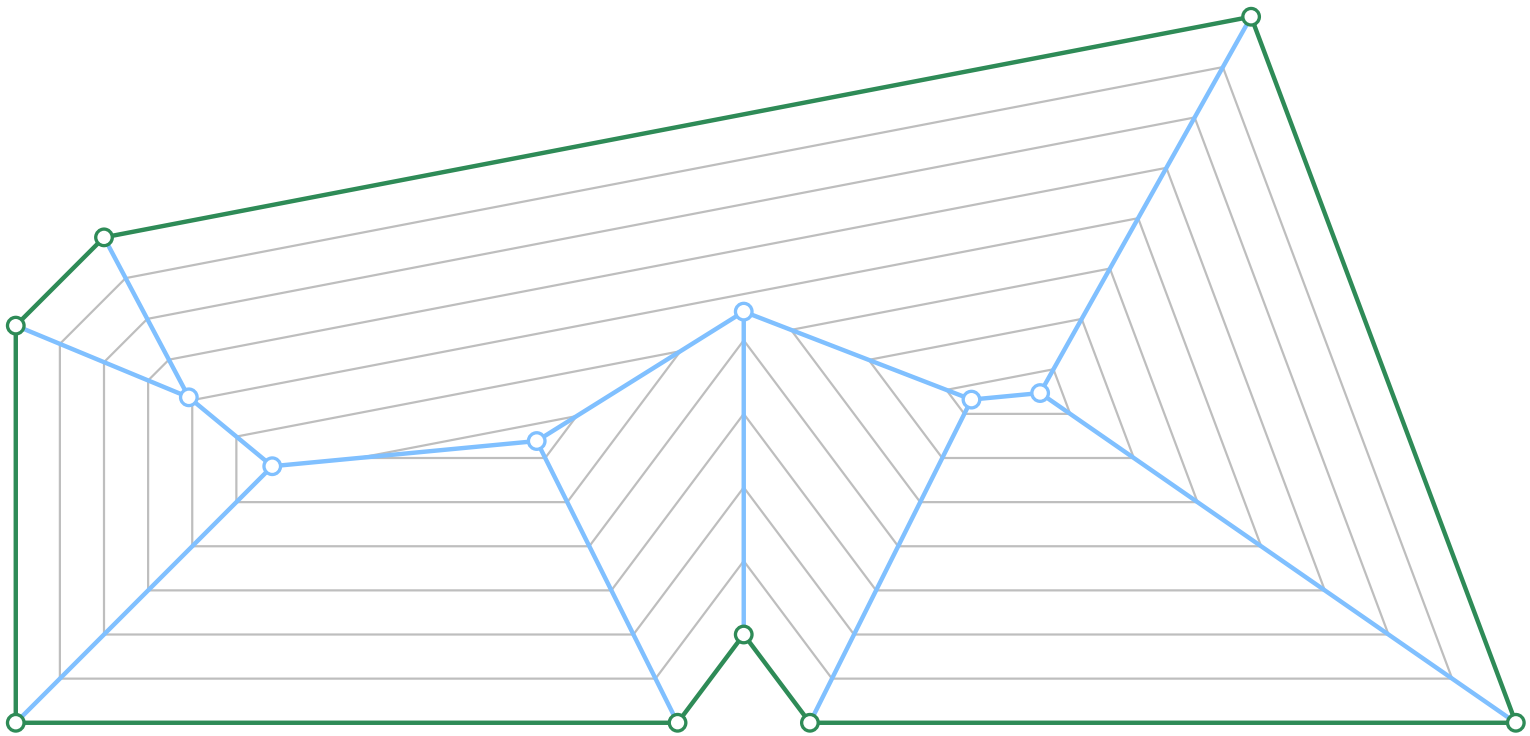

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- Collapsing triangles witness edge and split events.
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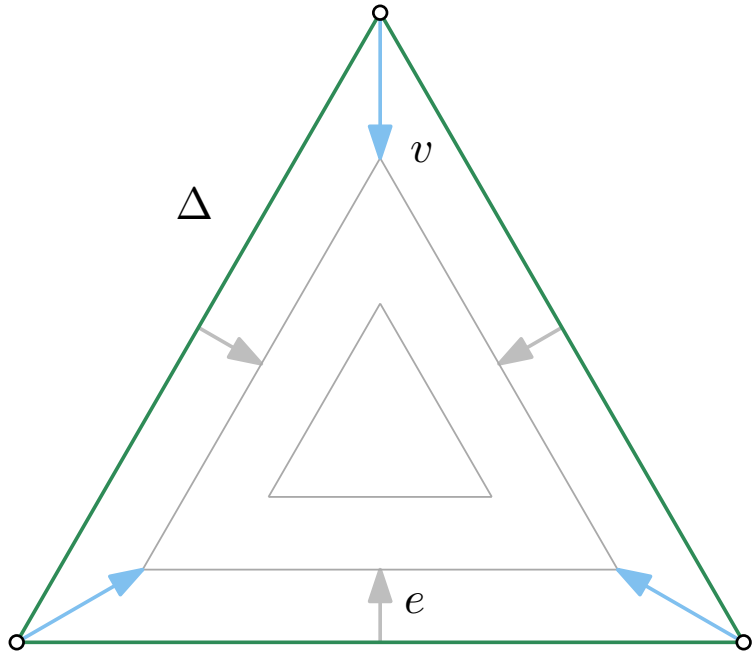

Triangulation-Based Algorithm

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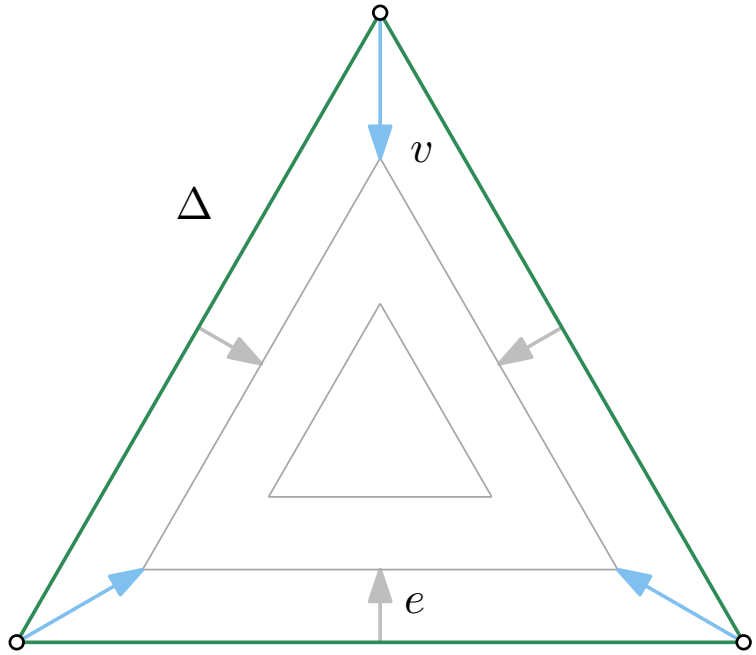
- That is, determine when the area of a triangle becomes zero.



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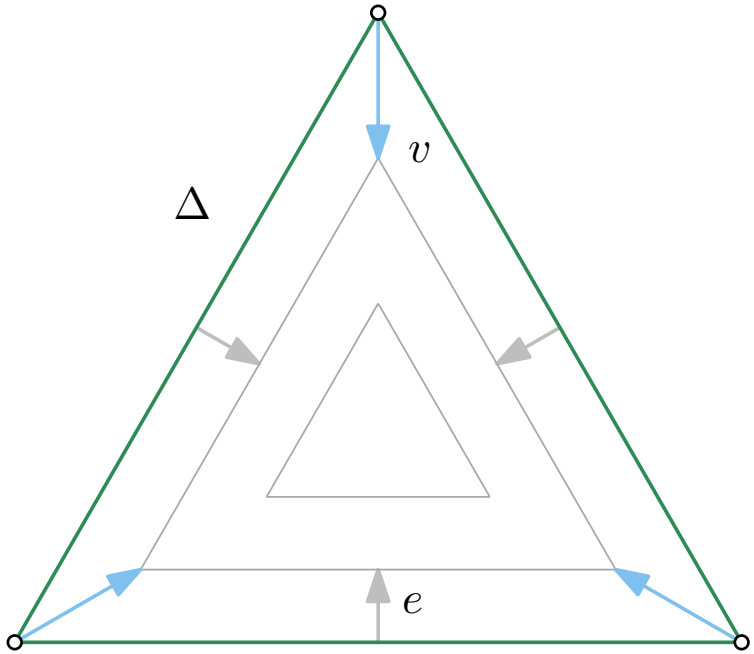

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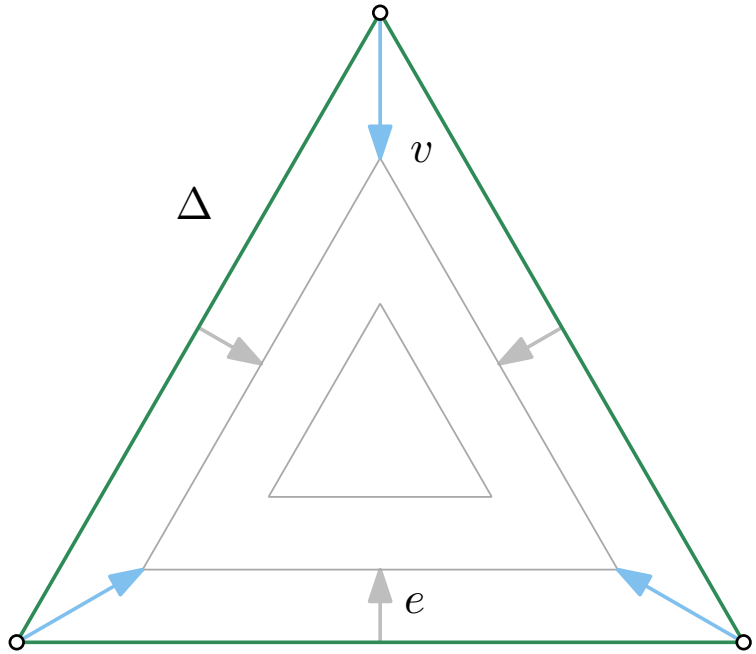

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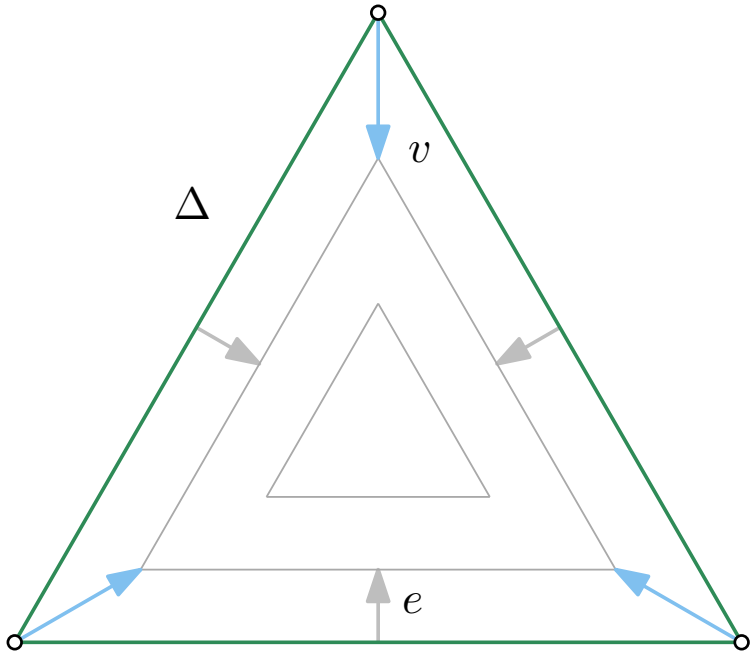

Algorithmic insight

Wavefront propagation based on kinetic triangulations allows to determine all events and to compute straight skeletons.

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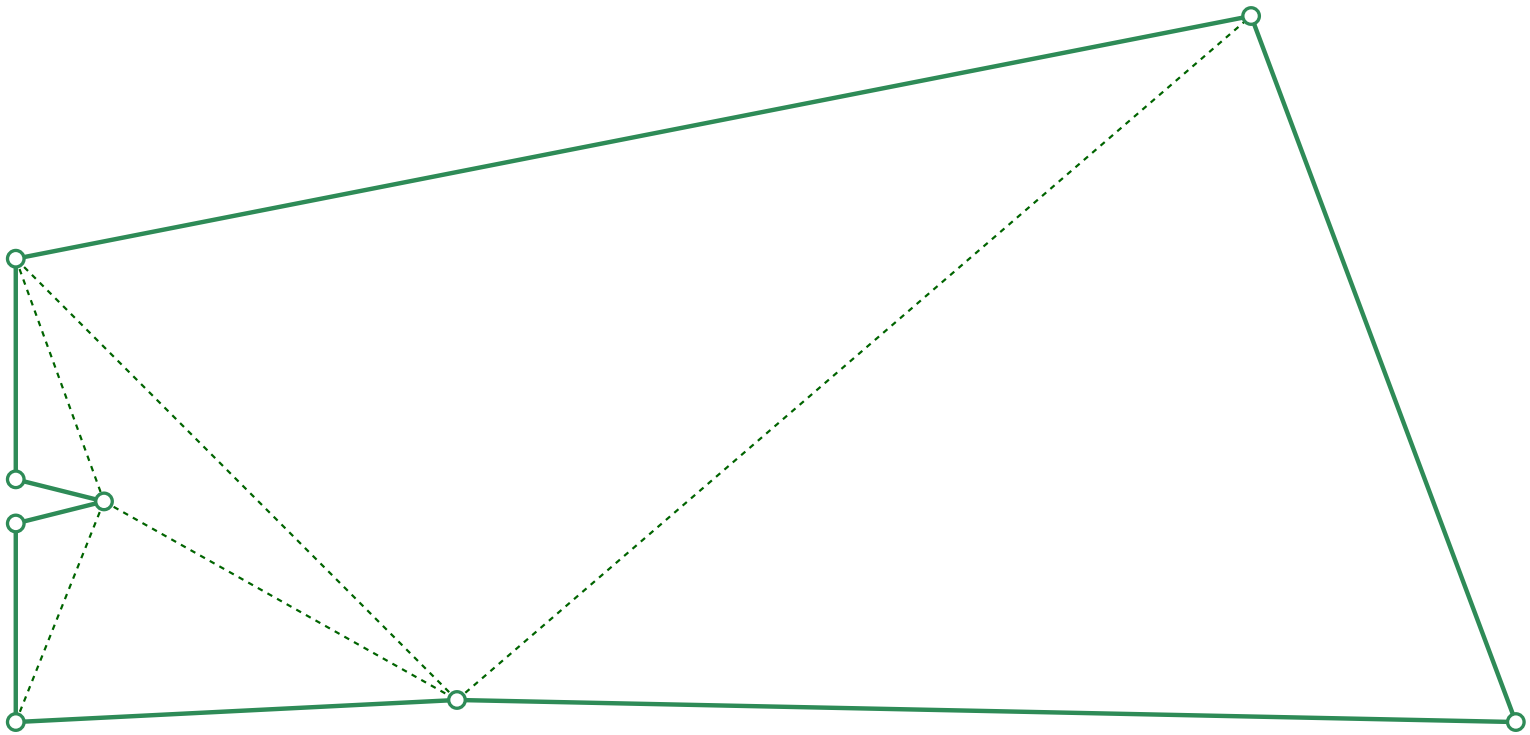
Triangulation-based Algorithm

Flip events

- Caveat: Not all collapses witness changes in the wavefront topology.



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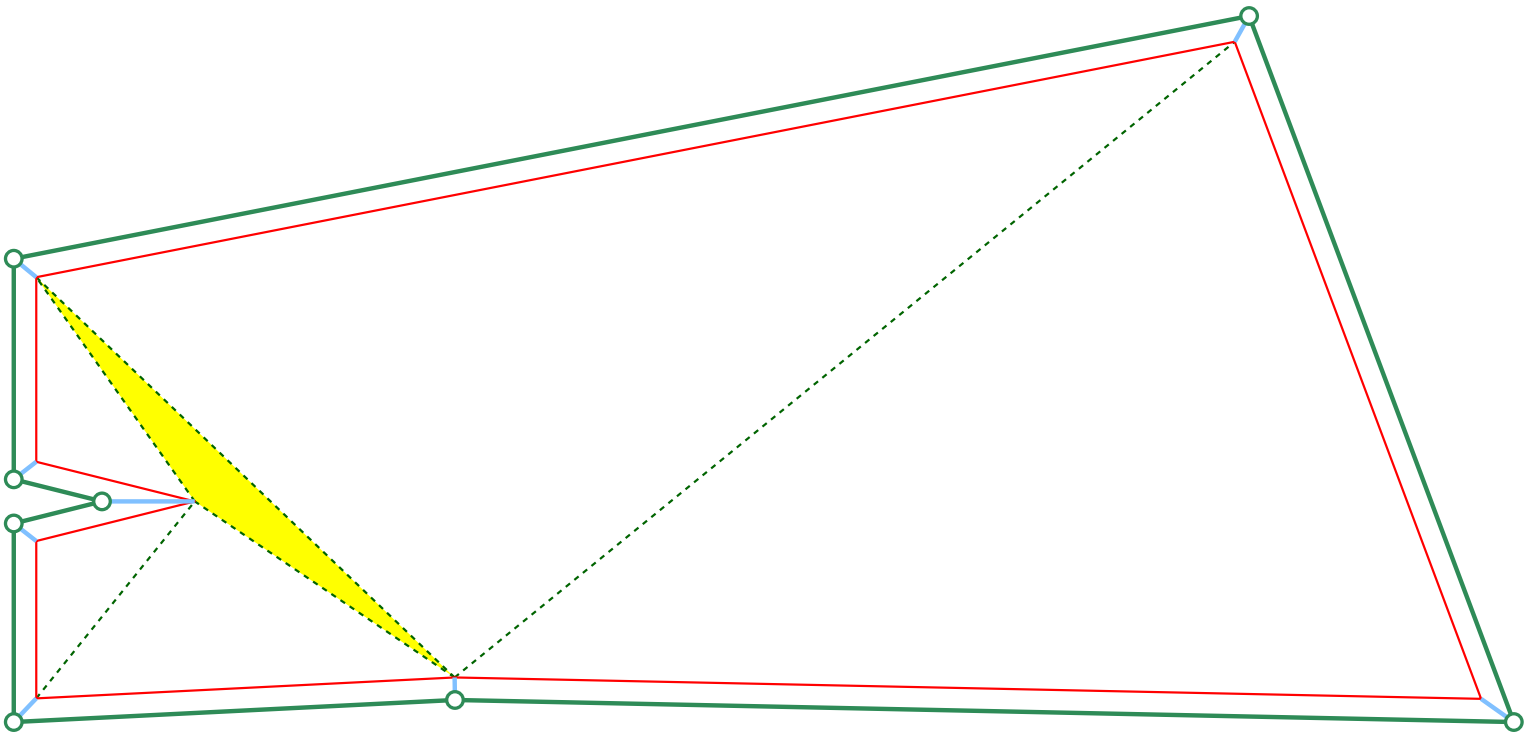

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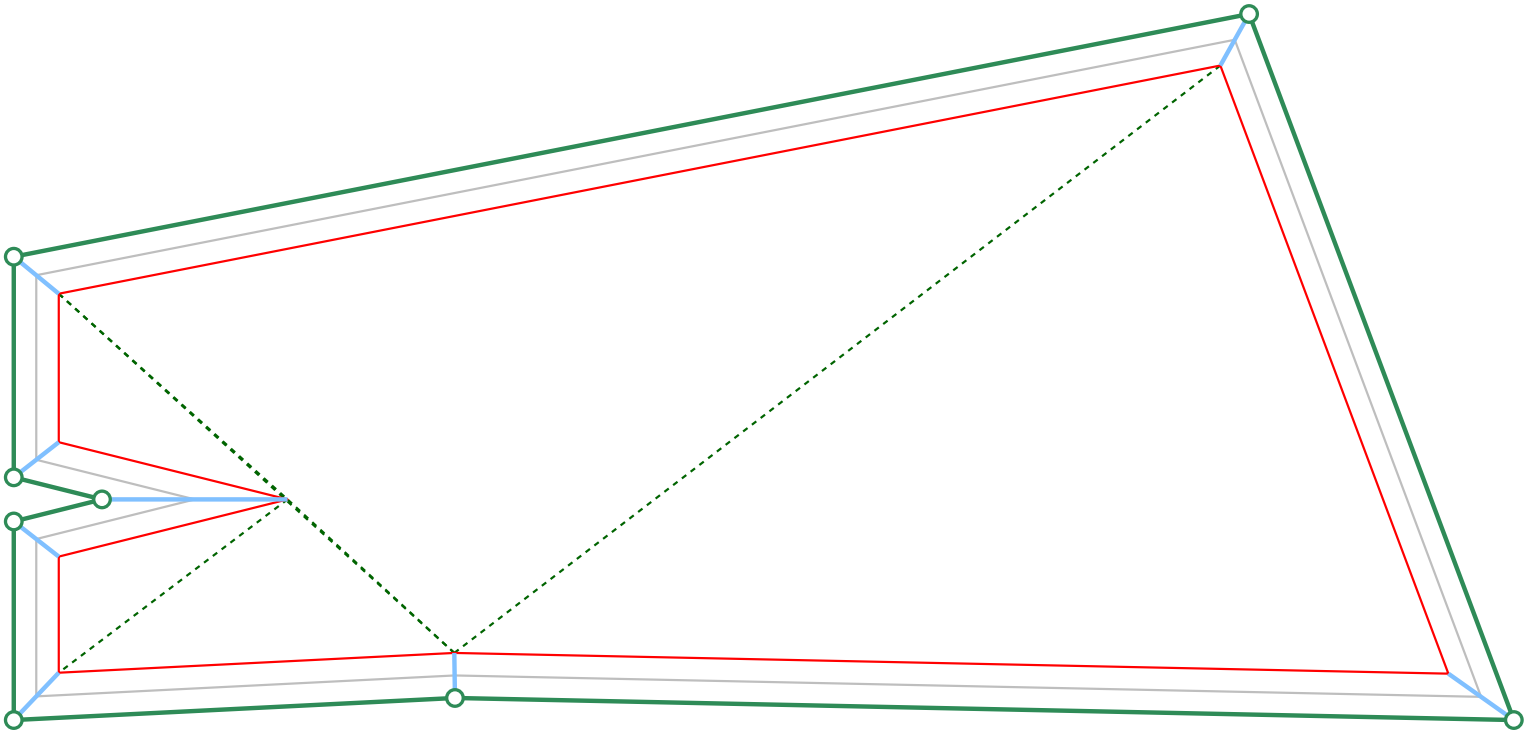

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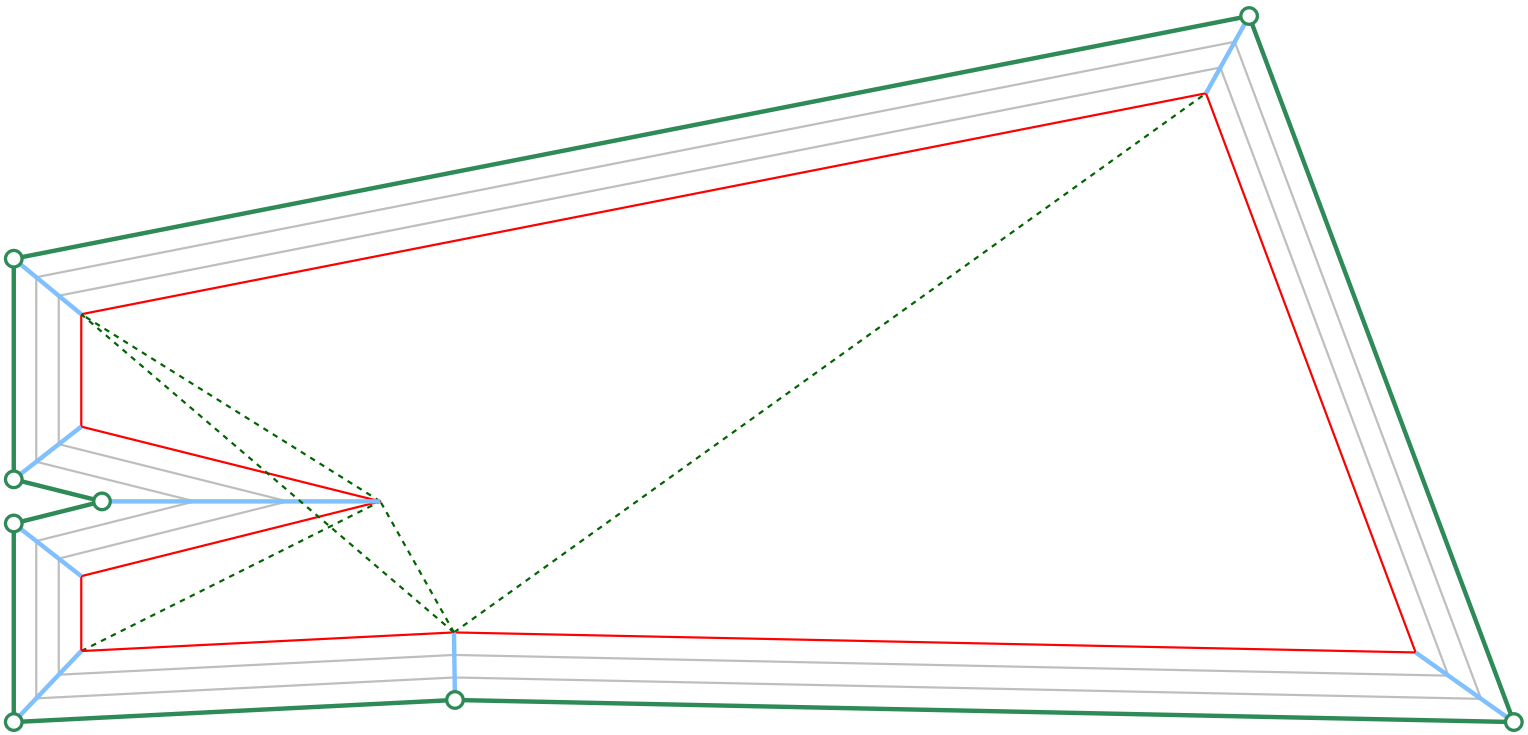

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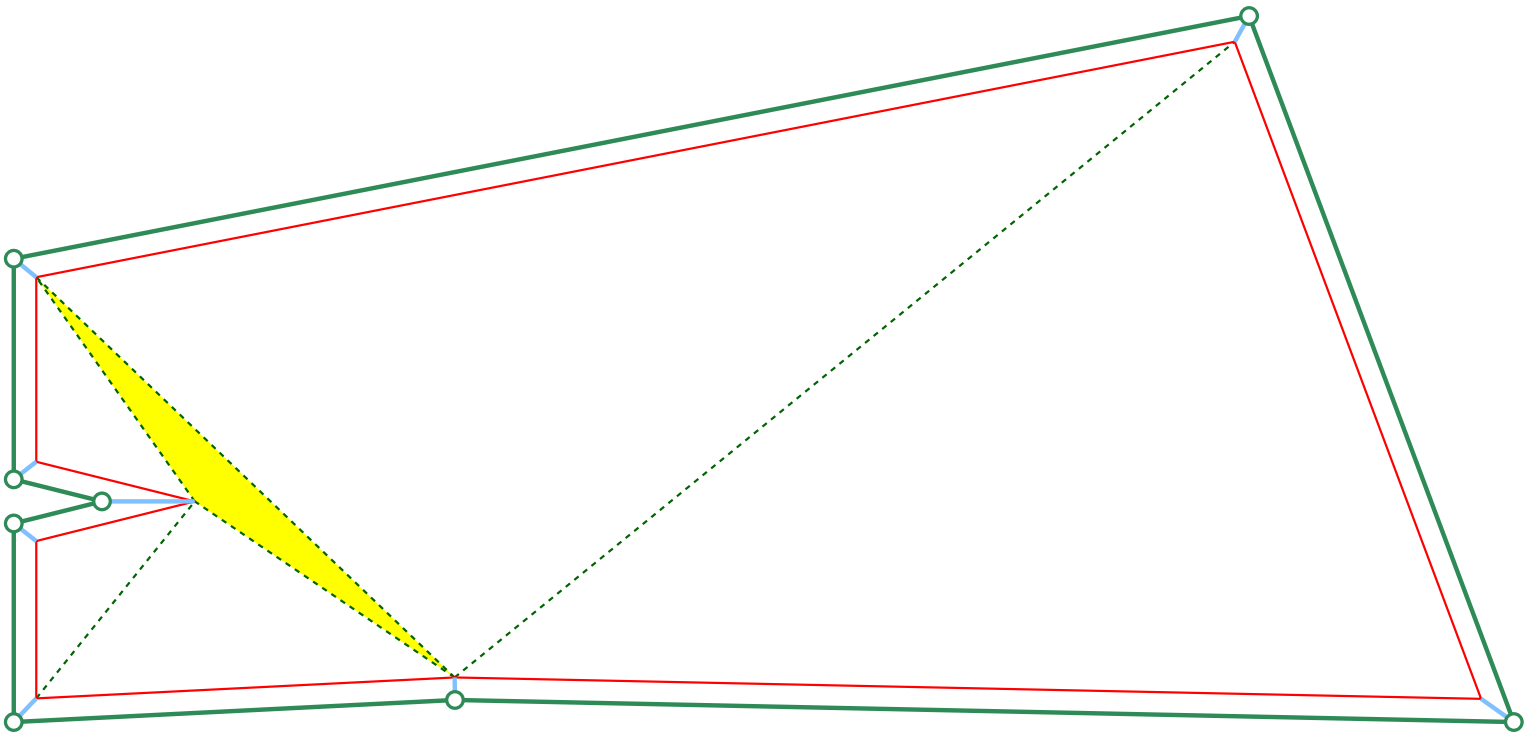

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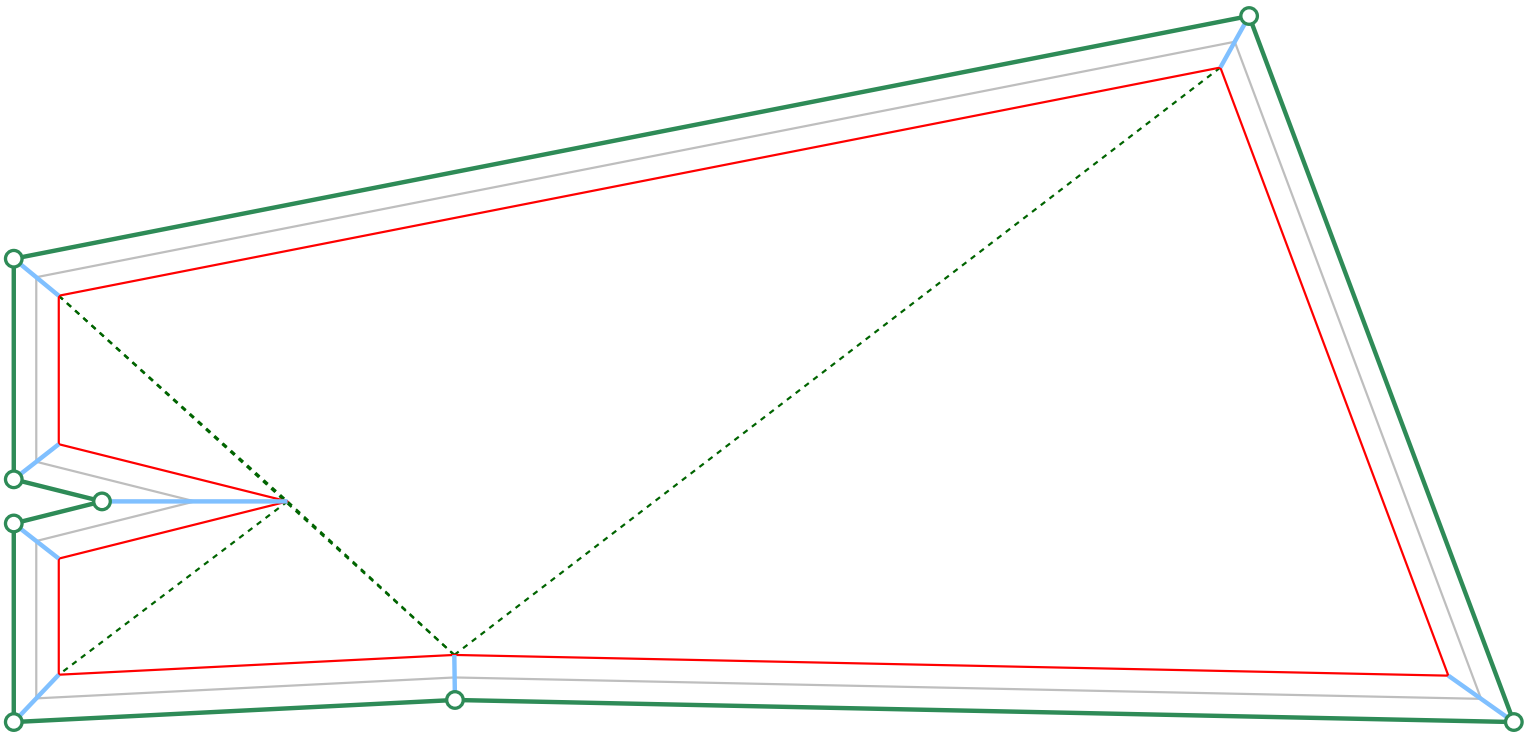

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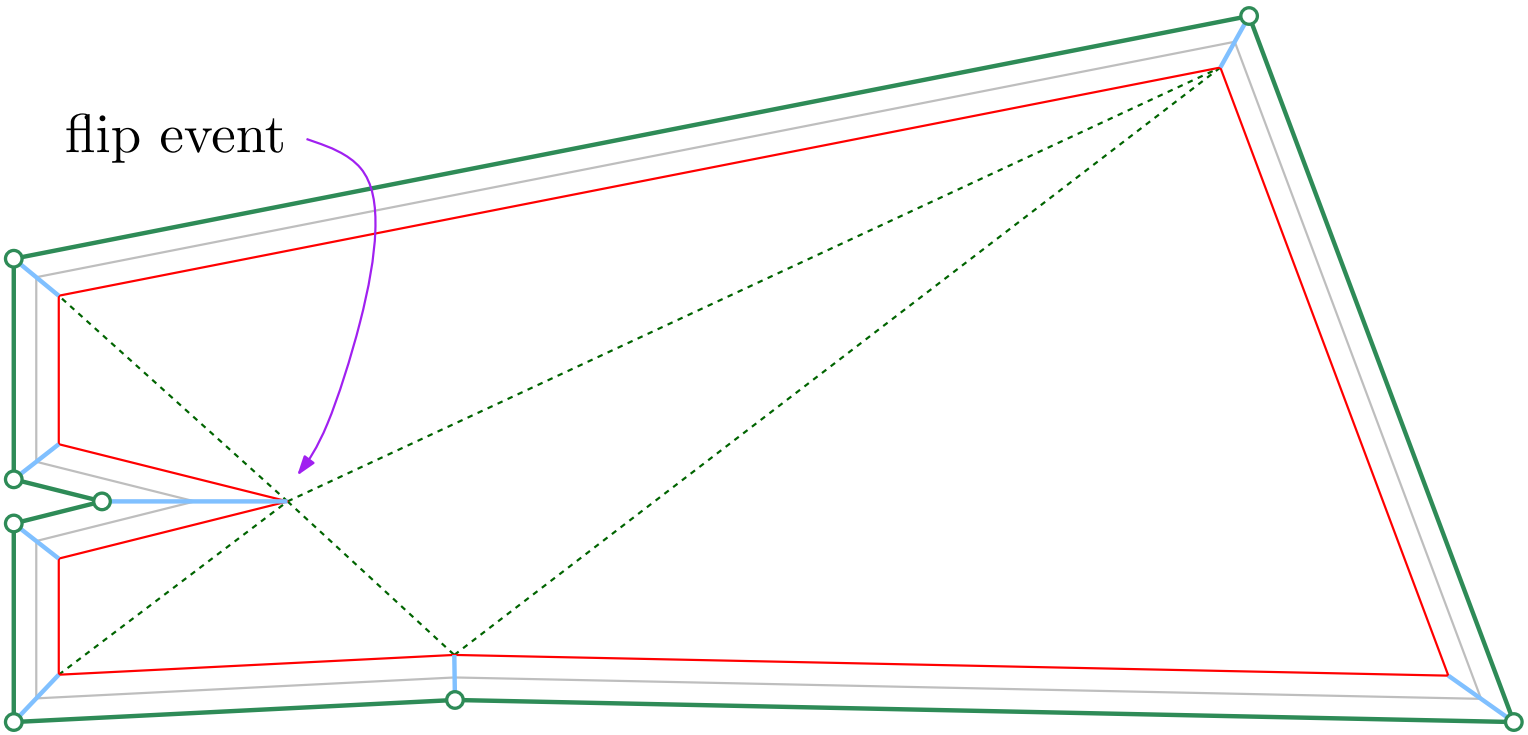

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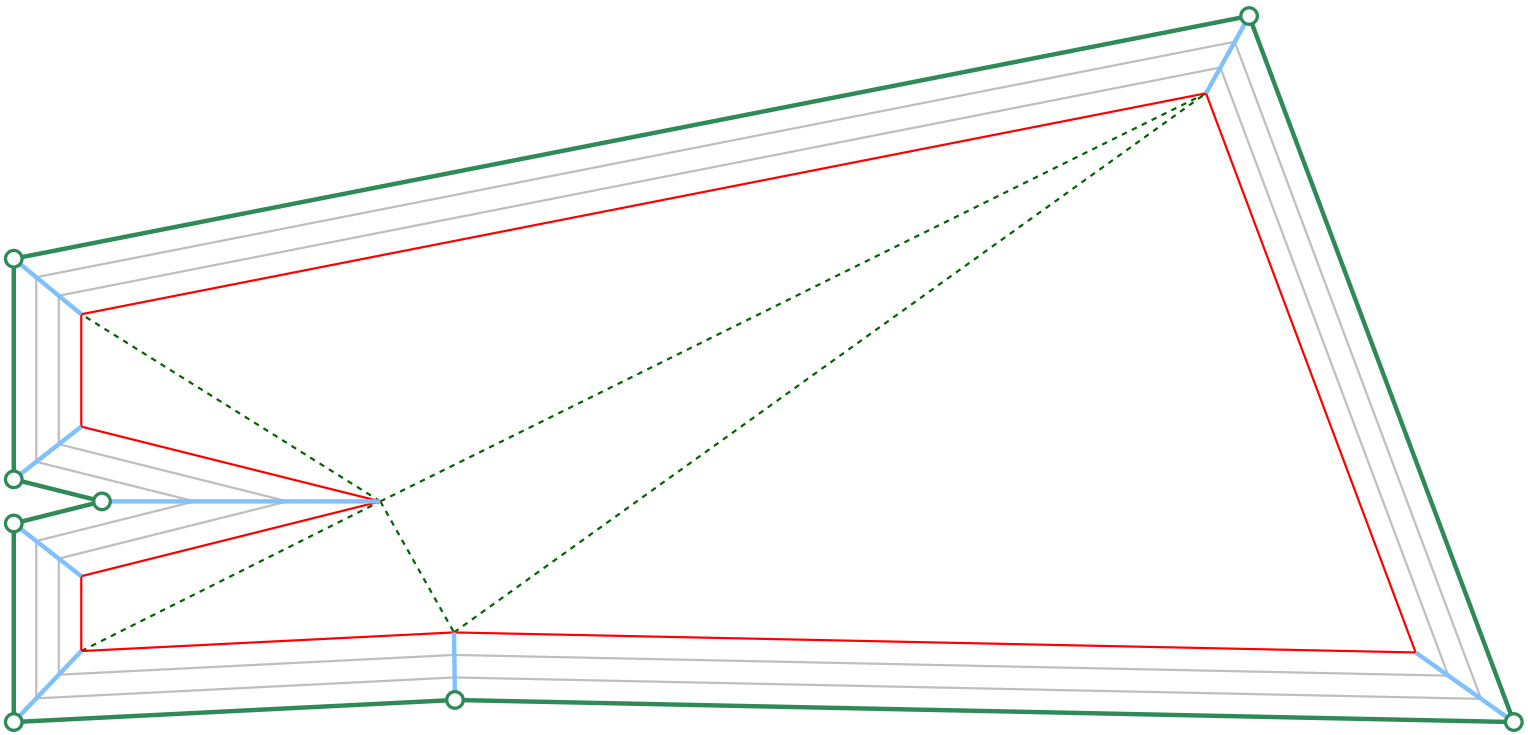

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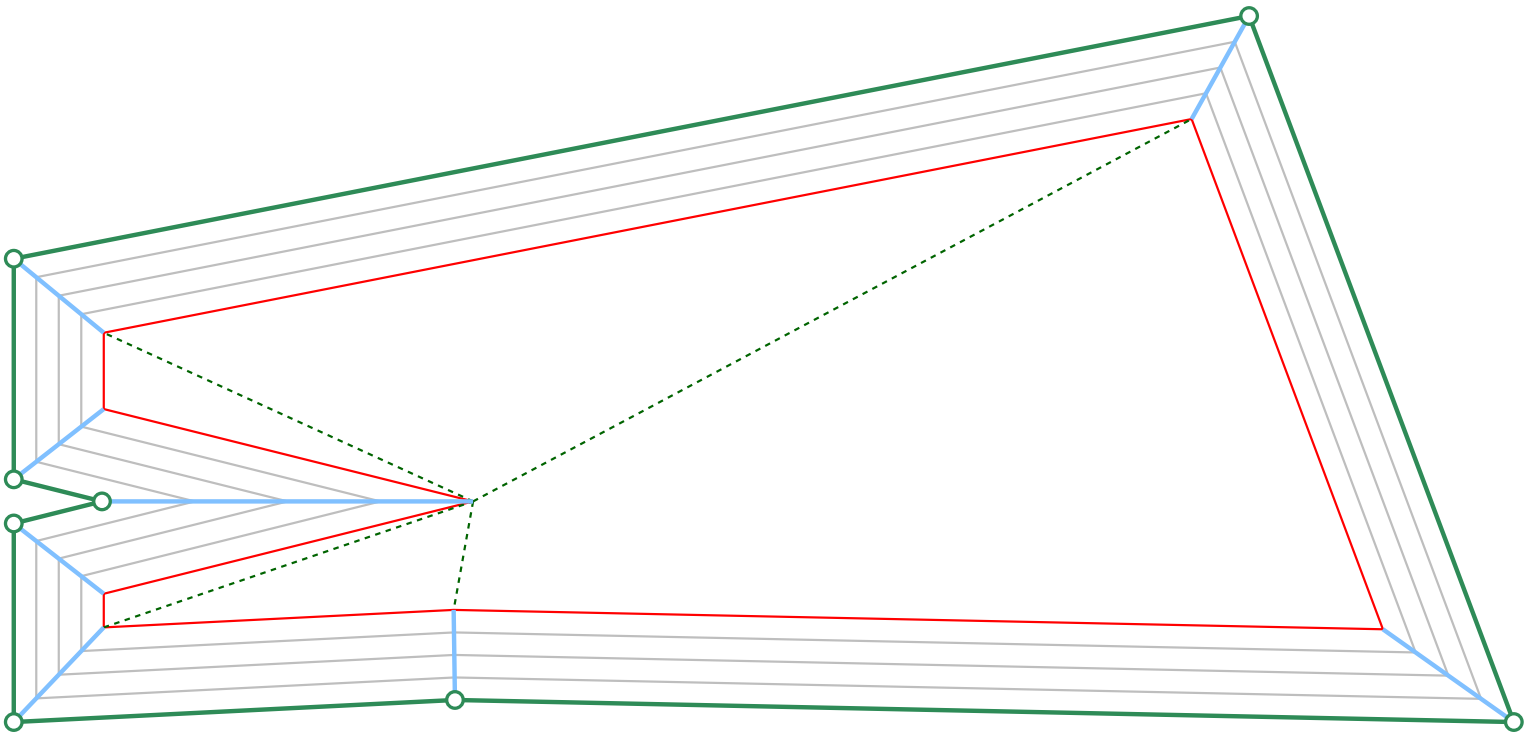

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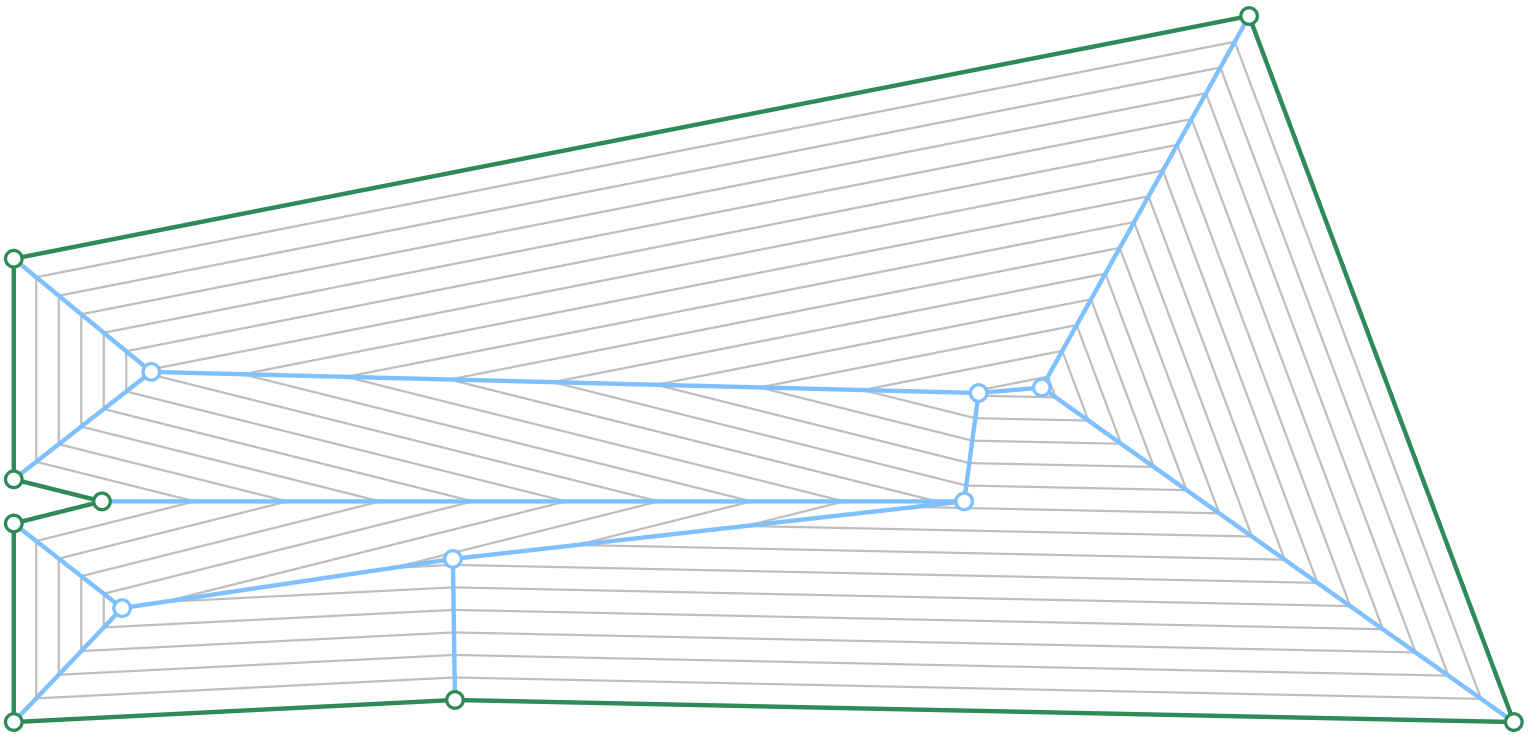

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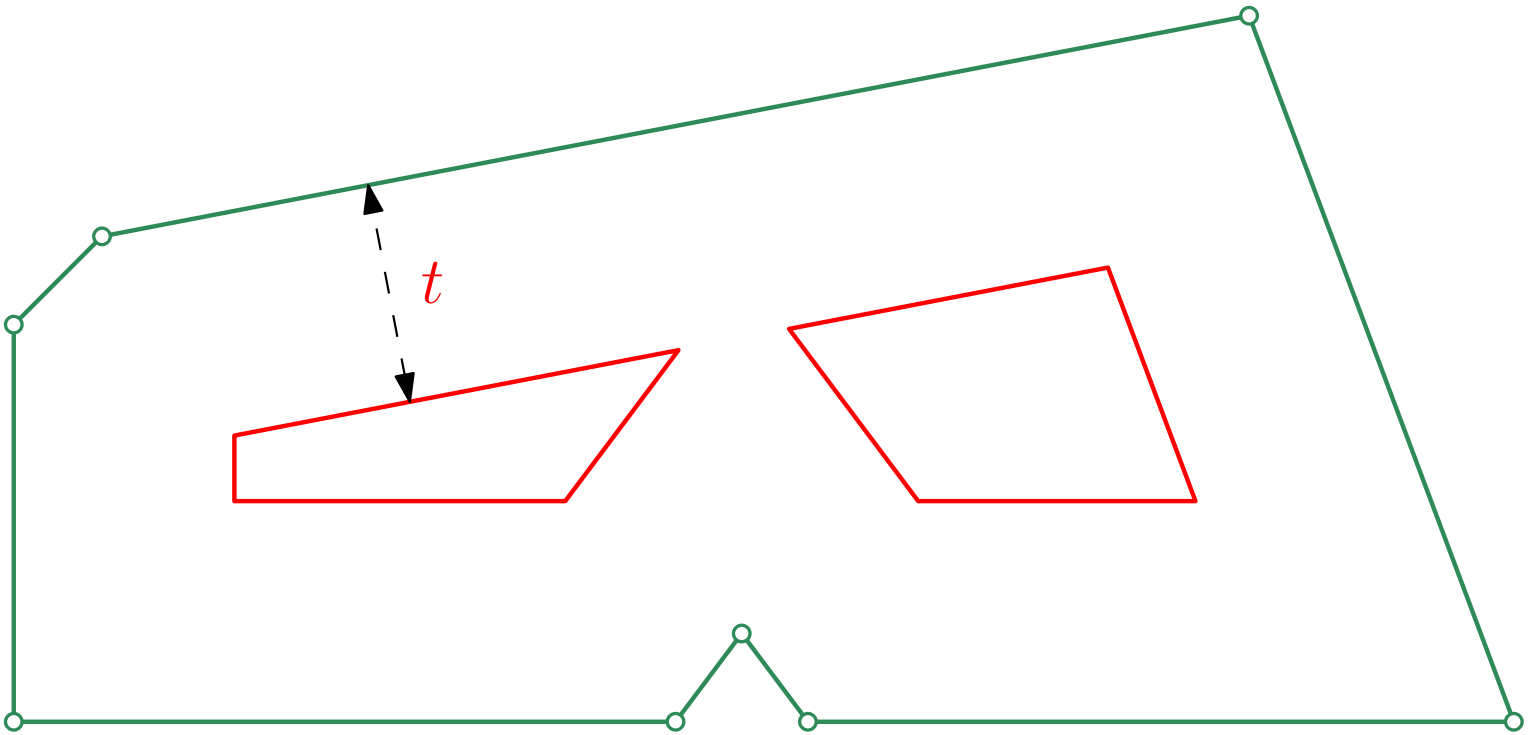
Offsetting

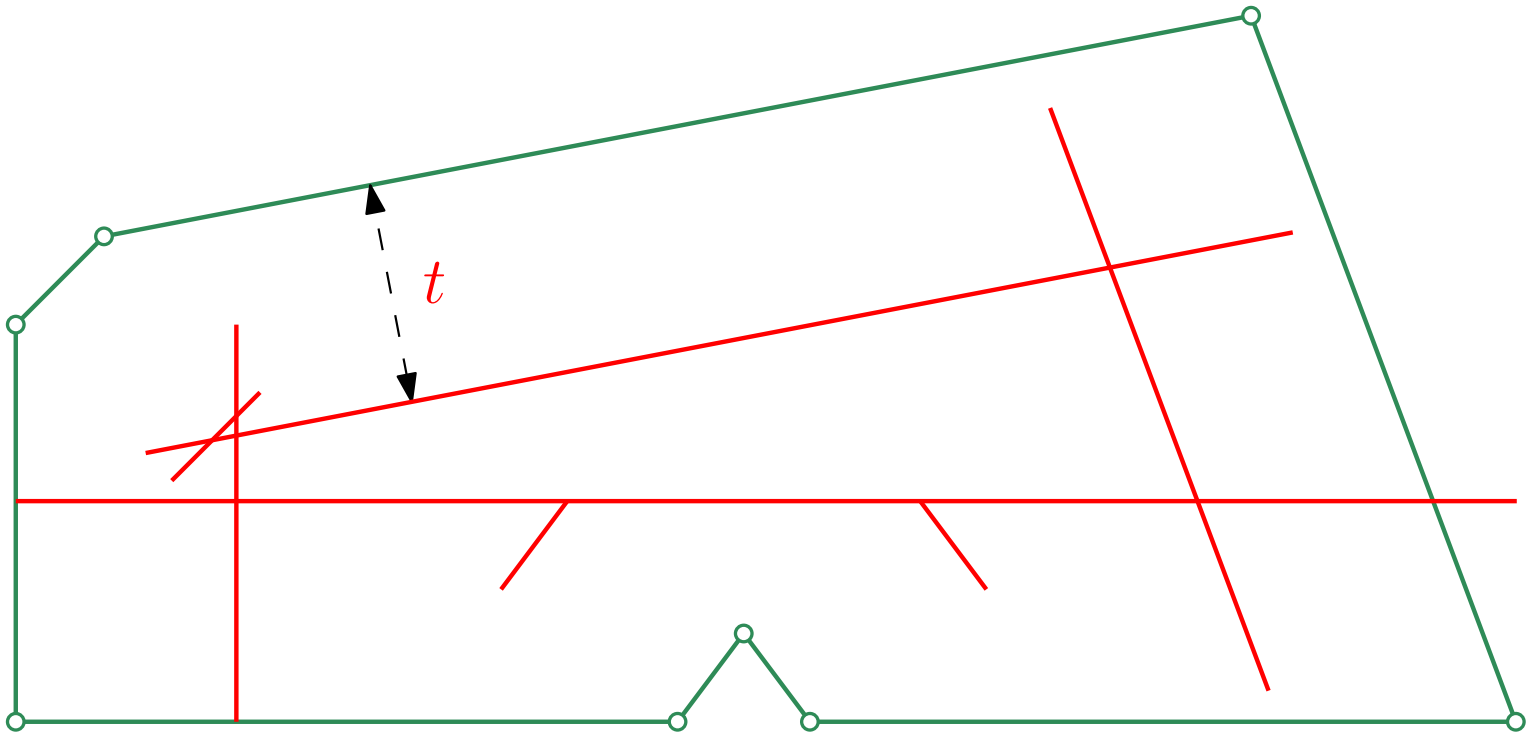
- How can we determine all offsets that correspond to some user-specified offset distance t?



Offsetting

Standard approach

1. Compute an elementary offset segment for each input segment.



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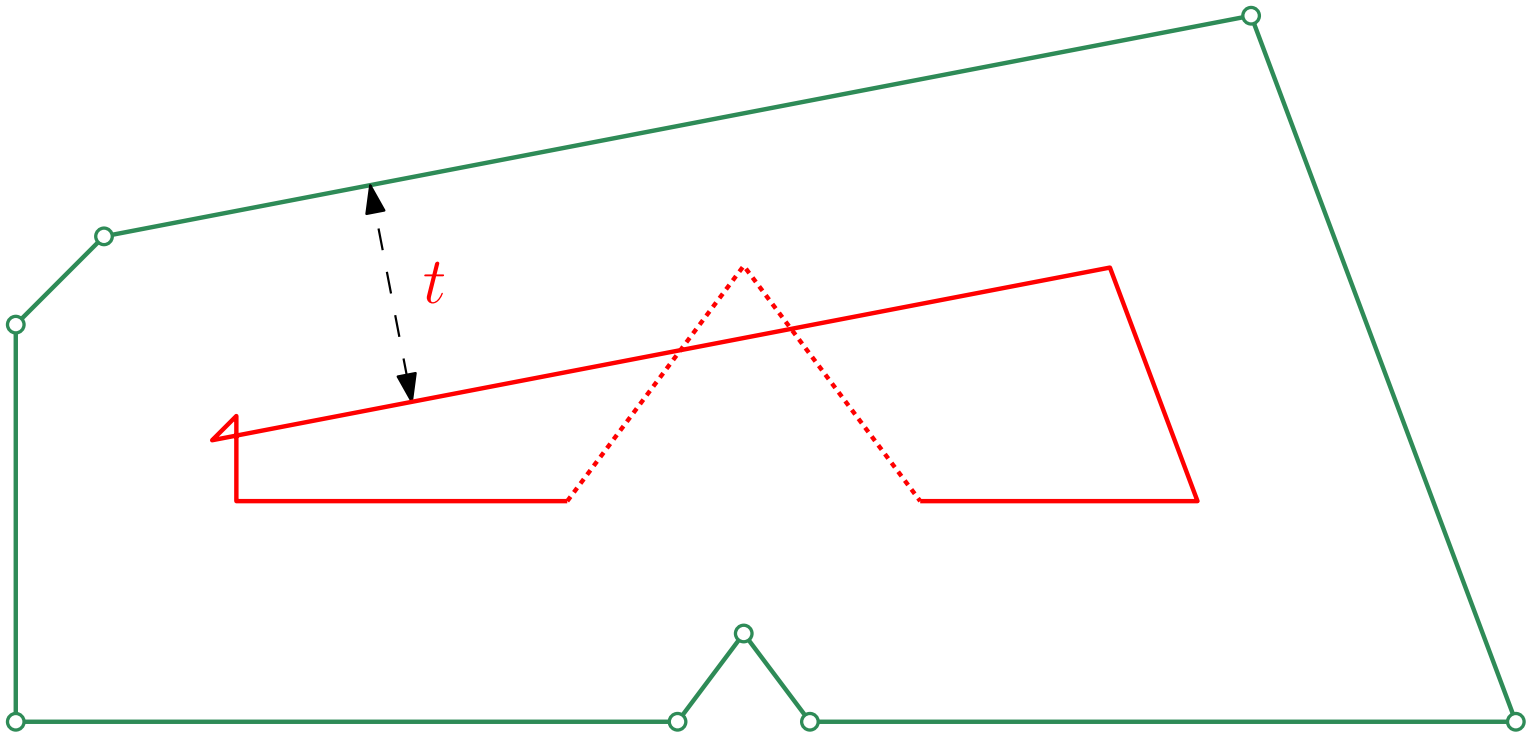
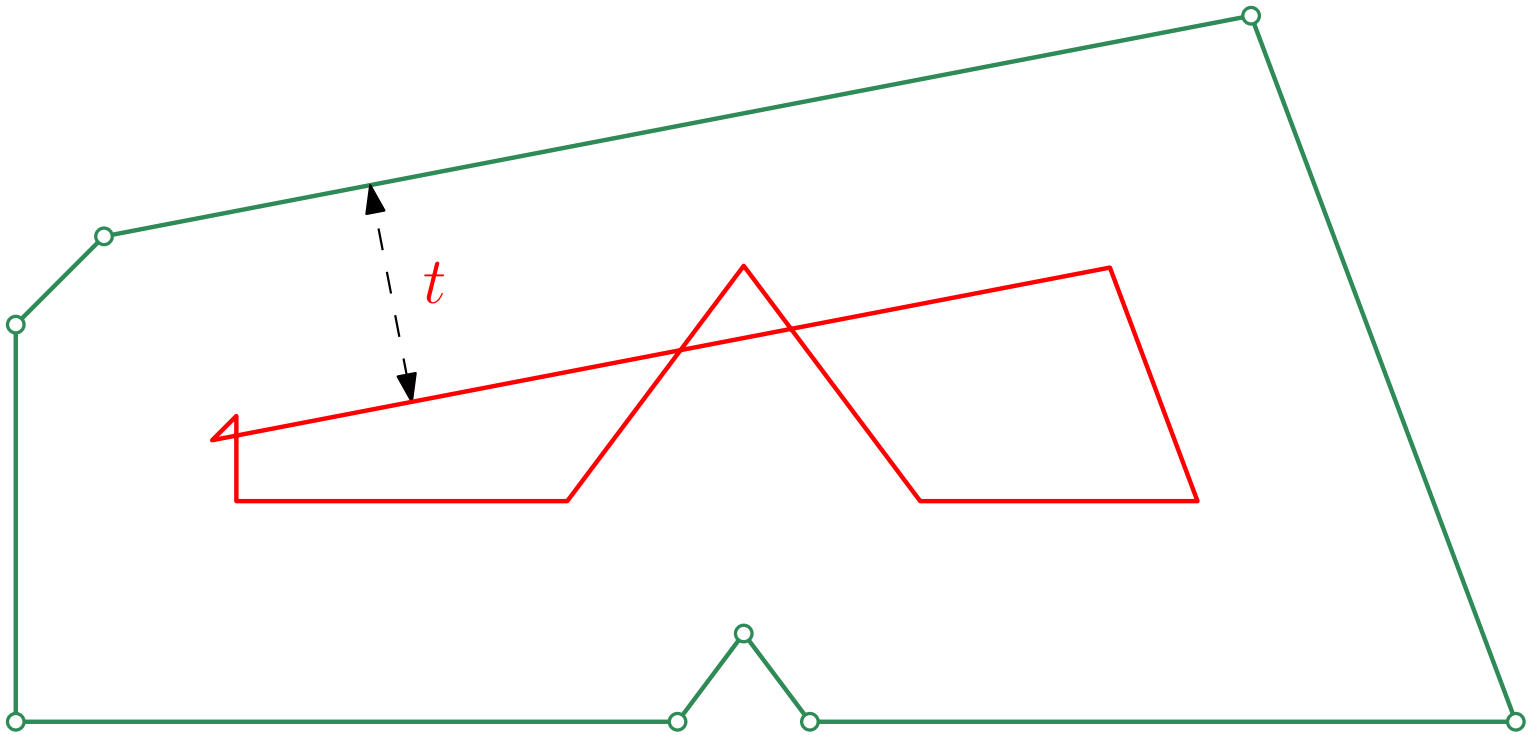
1. Compute an elementary offset segment for each input segment.
2. Trim at intersections of neighboring segments,



Offsetting

Standard approach

1. Compute an elementary offset segment for each input segment.
2. Trim at intersections of neighboring segments, and close gaps to form one loop.



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1. Compute an elementary offset segment for each input segment.
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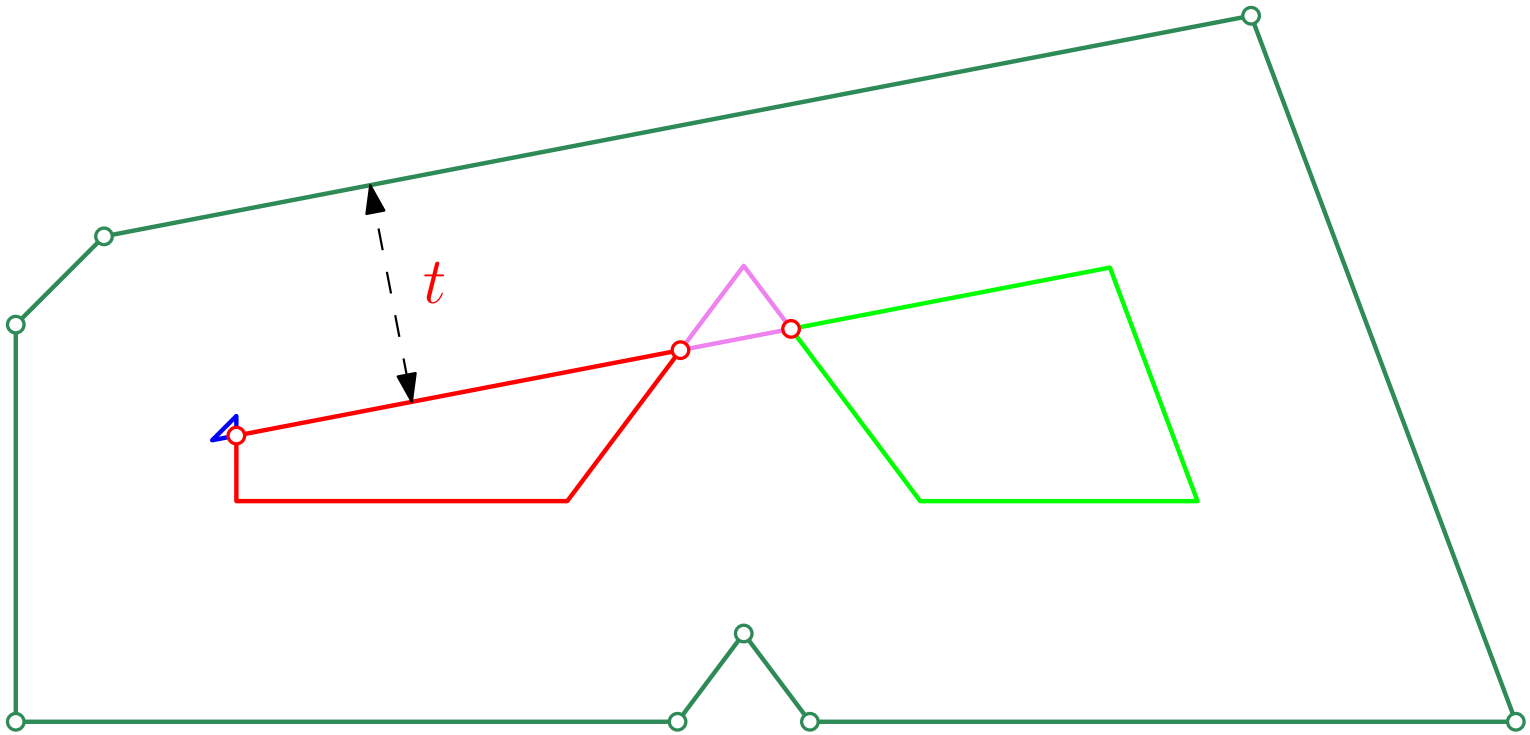
3. Determine all self-intersections and split into several loops.



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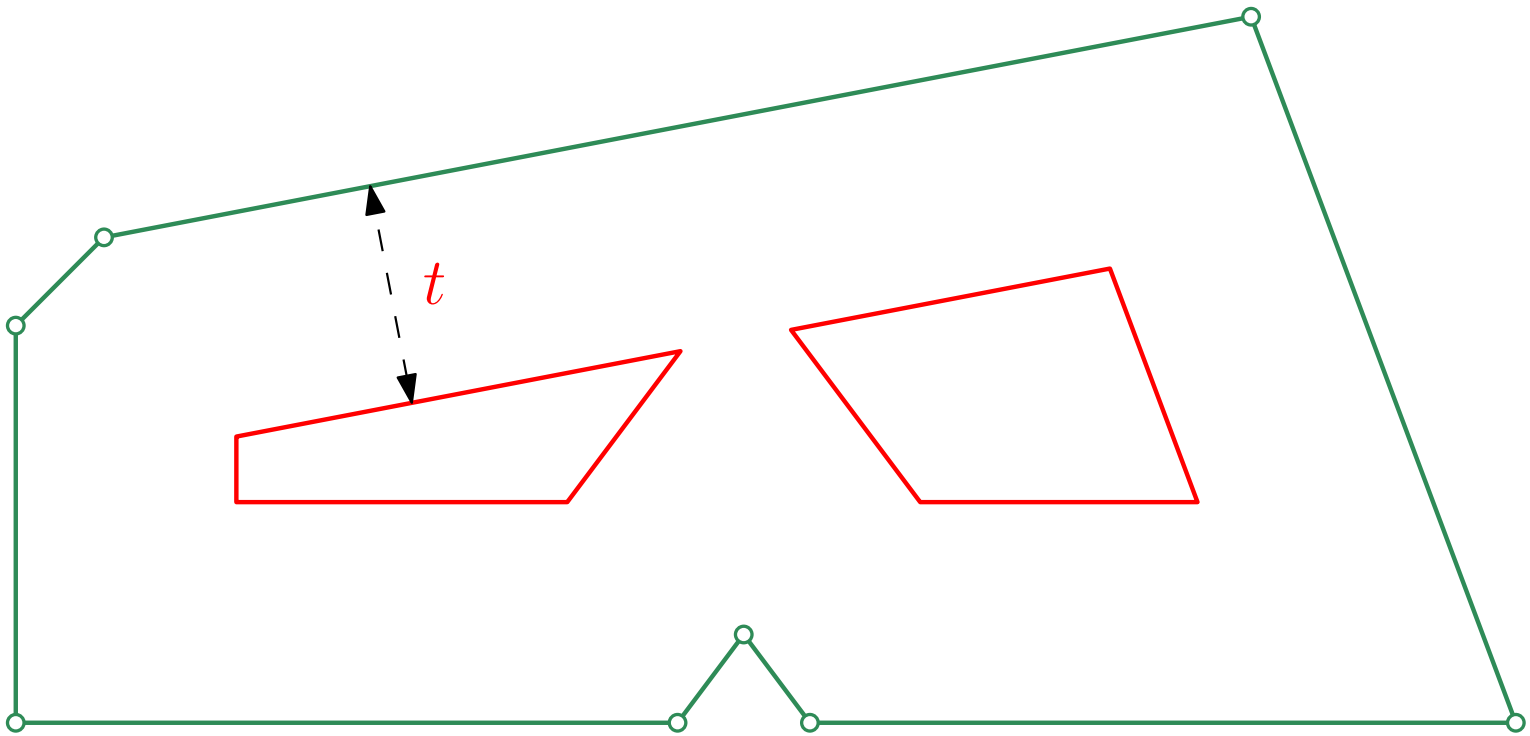
3. Determine all self-intersections and split into several loops.
4. Discard excess loops.



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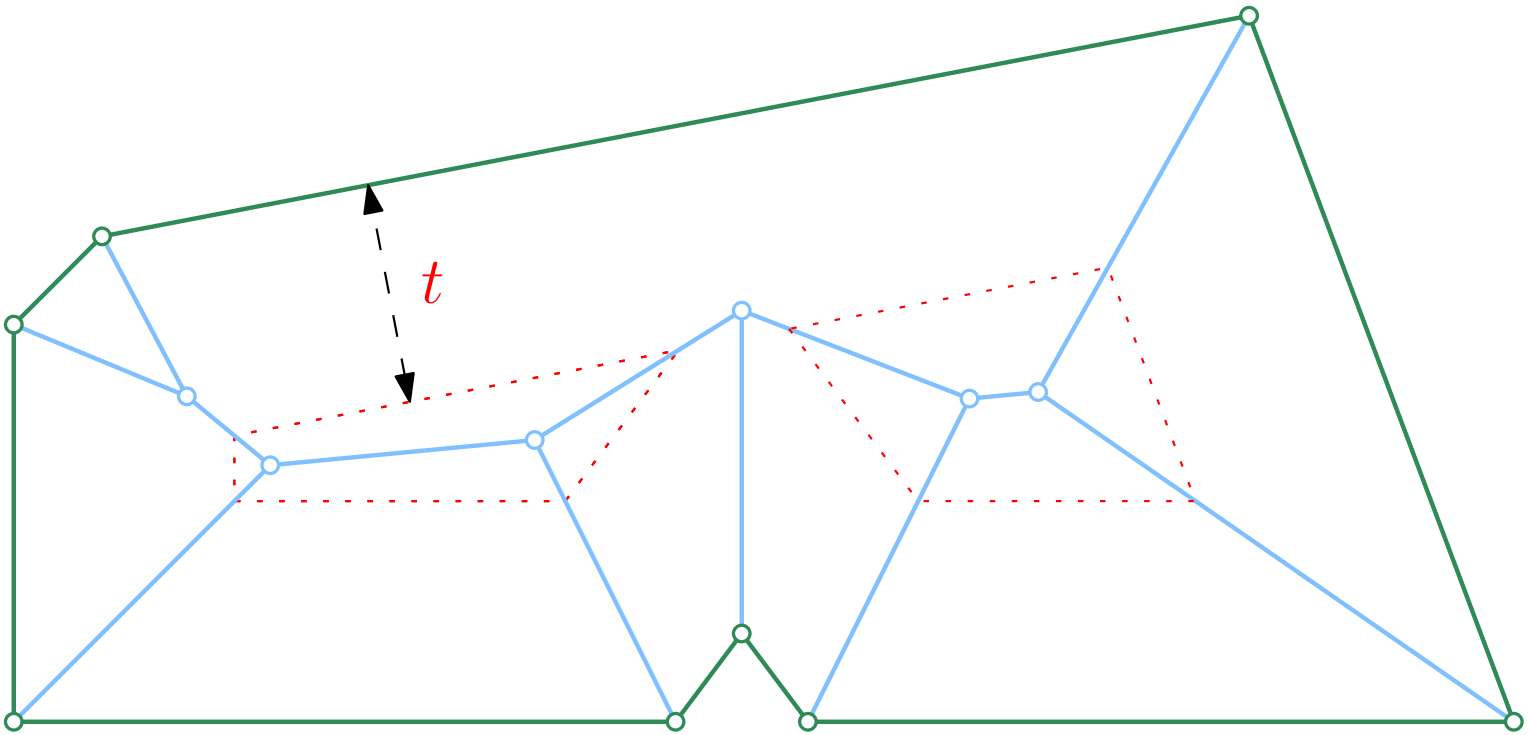
Offsetting Based on Straight Skeleton



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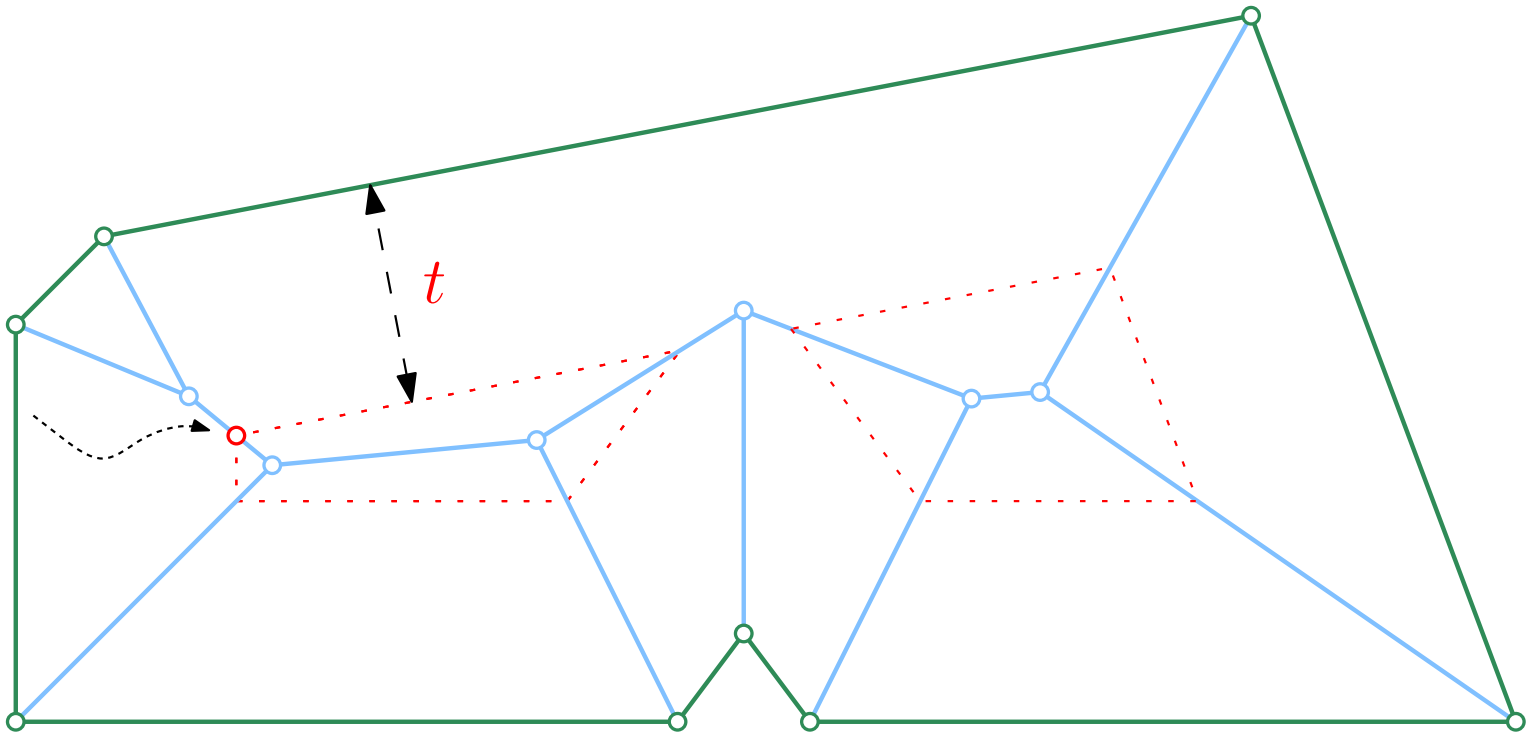
Offsetting Based on Straight Skeleton

Scan straight skeleton

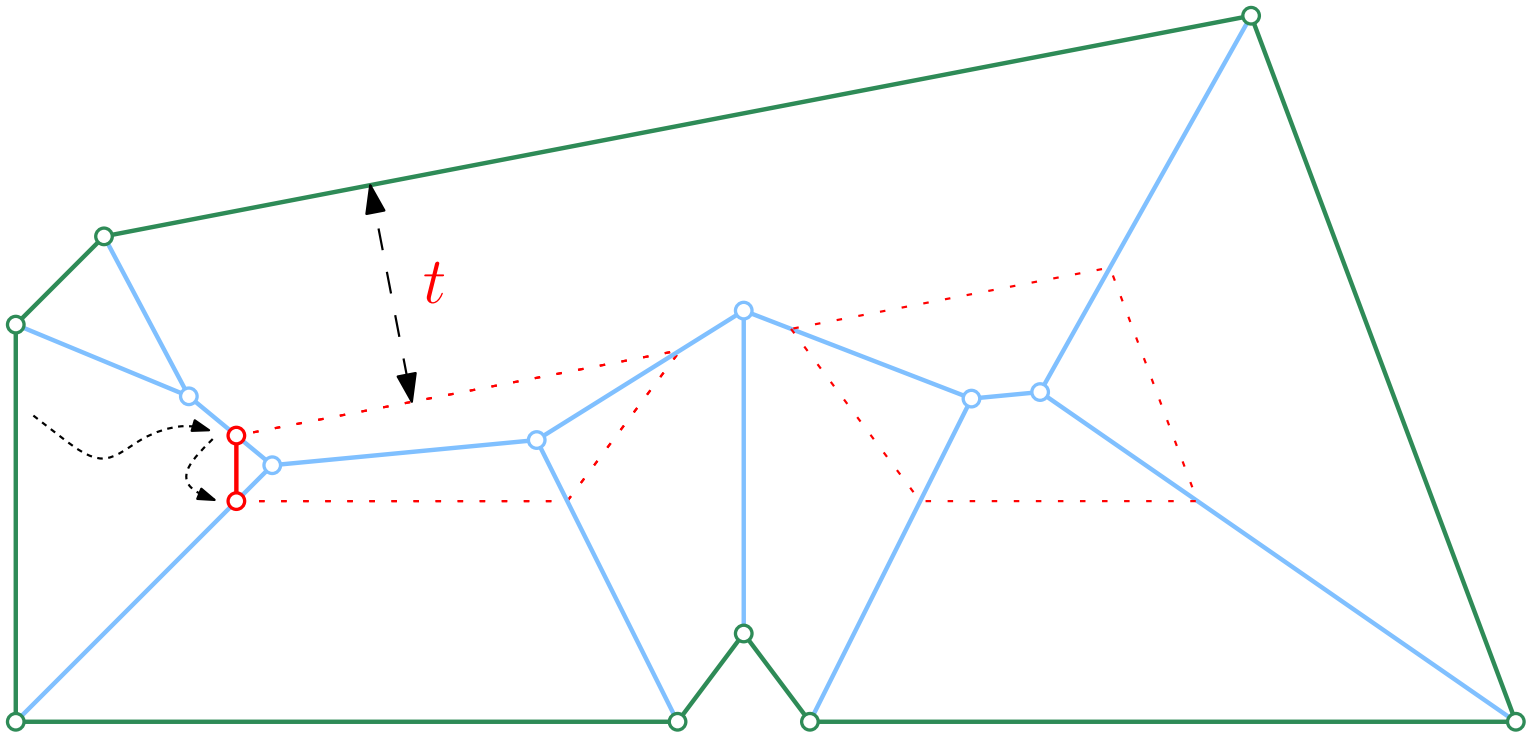
1. Choose SK edge not yet intersected by an offset loop; compute start vertex.



Offsetting Based on Straight Skeleton

Scan straight skeleton

1. Choose SK edge not yet intersected by an offset loop; compute start vertex.
2. Advance clockwise along boundary of SK face and compute next vertex.



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Offsetting Based on Straight Skeleton

Scan straight skeleton

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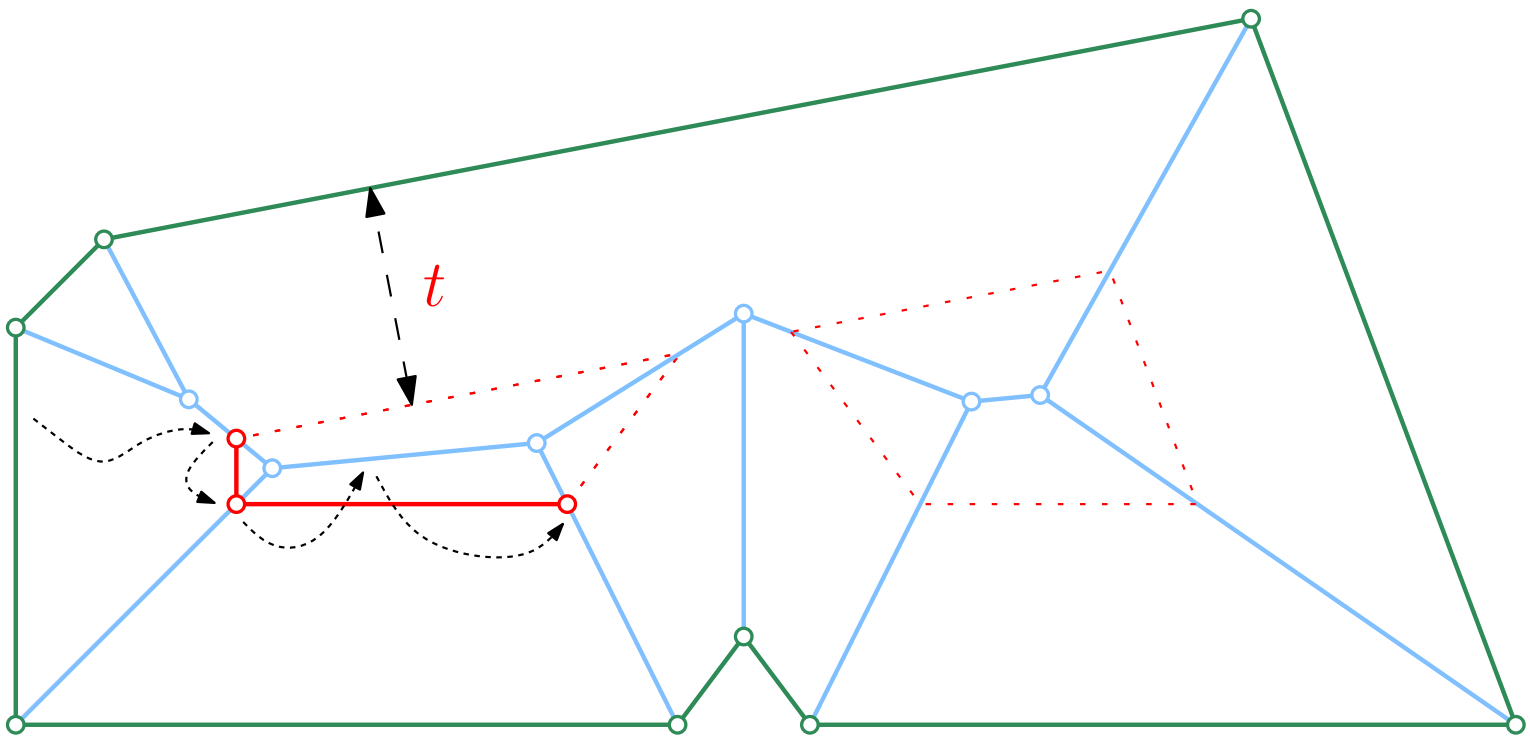

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Scan straight skeleton

1. Choose SK edge not yet intersected by an offset loop; compute start vertex.
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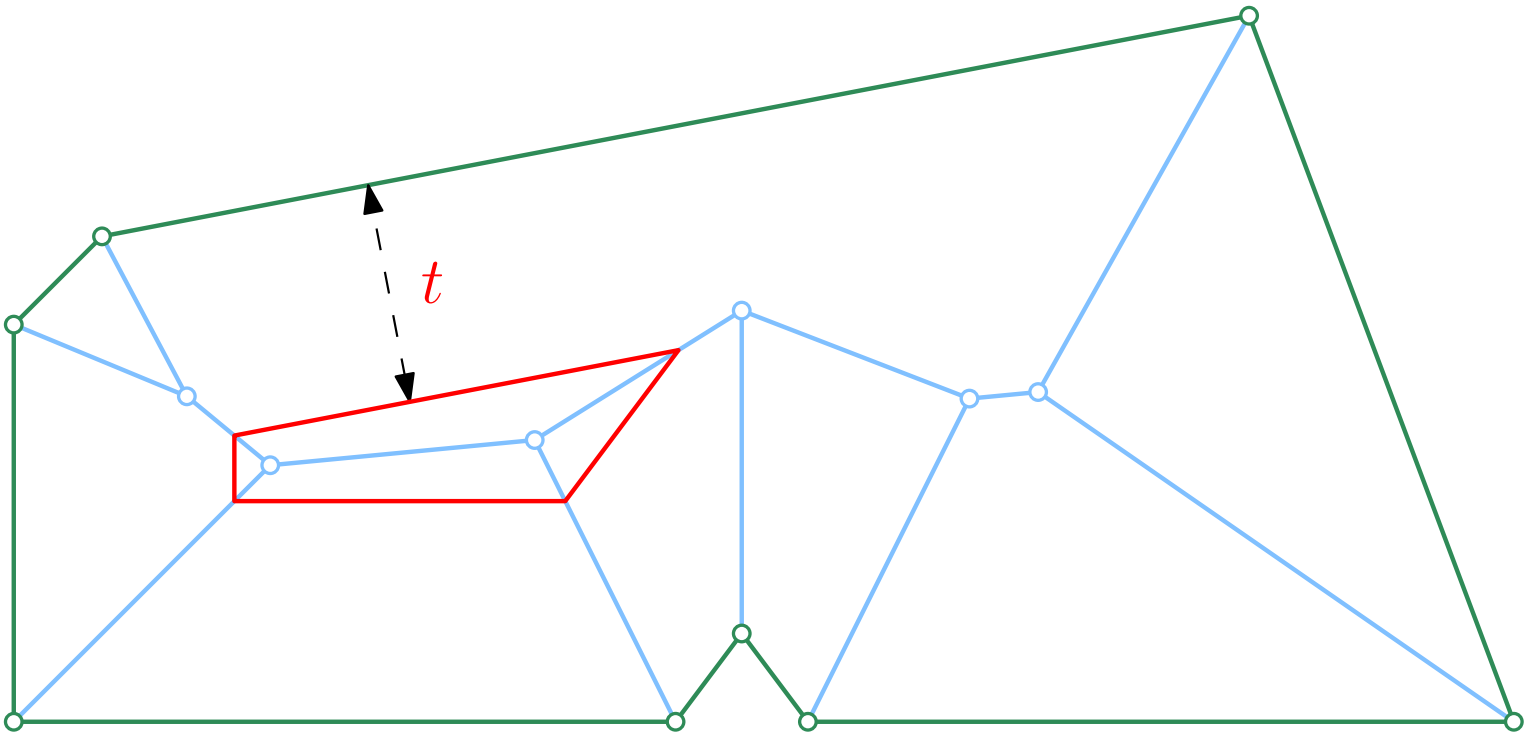
3. Move to neighboring face and keep scanning that face clockwise.
4. Finish one offset curve.



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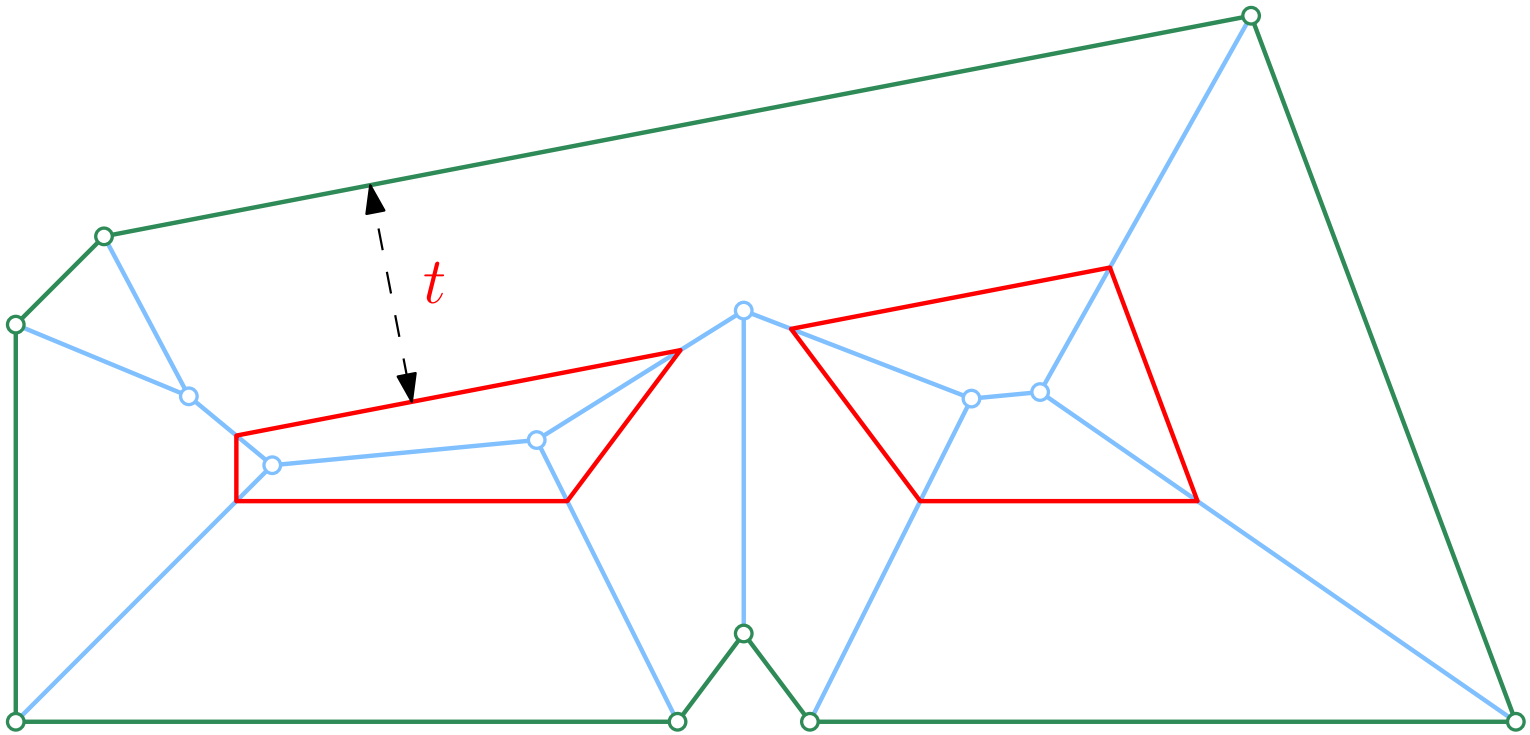
4. Finish one offset curve. Continue with next offset curve.



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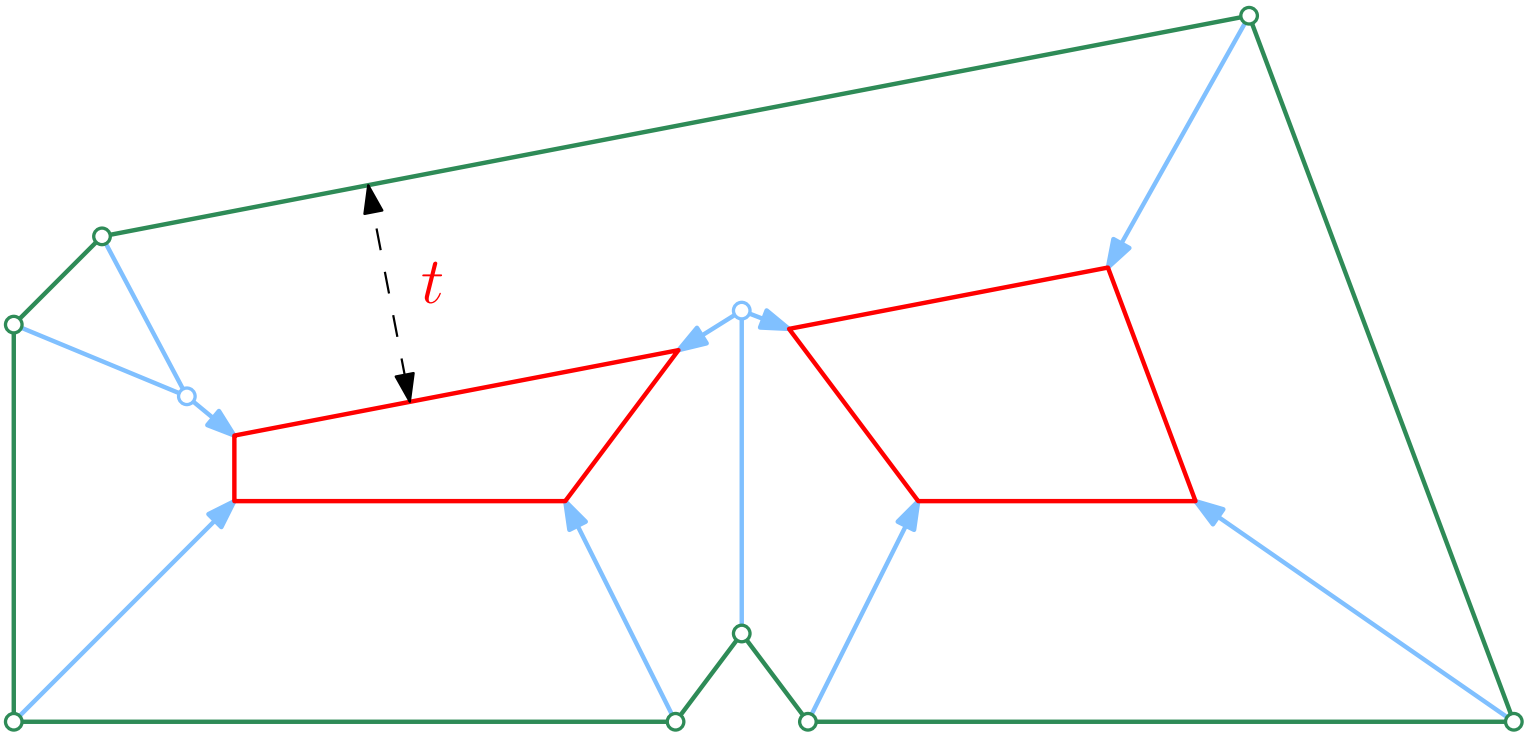
Offsetting Based on Straight Skeleton

Alternative: Halt wavefront

Halt wavefront-propagation when the offset distance t is reached.



Implementation

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- Need to handle degeneracies that cause multiple simultaneous events [P.&H.].
- Need to detect and classify simultaneous events reliably on a standard floating-point arithmetic [P.&H.].

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SURFER

Straight-skeleton algorithm, based on kinetic triangulations, implemented in C and named SURFER.

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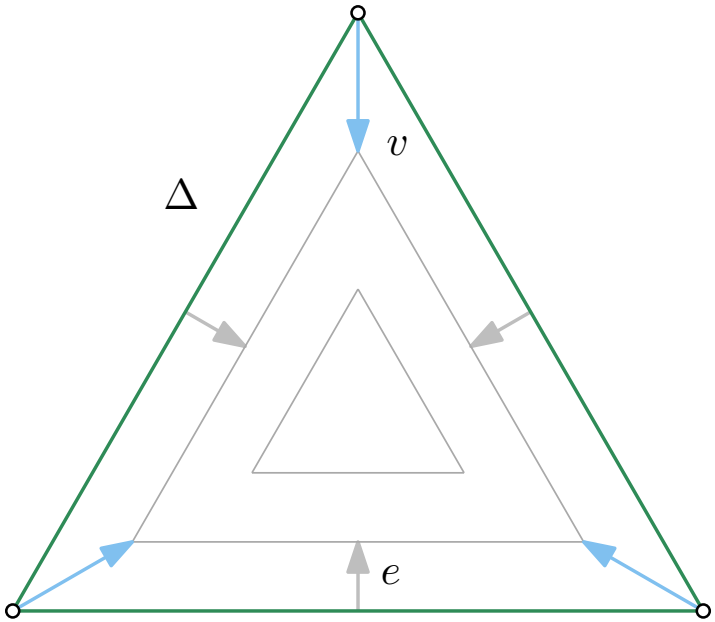
Implementation: Finding and Classifying Collapse Times

Different ways to compute collapse time

Suppose that the three vertices of a triangle move towards one point.



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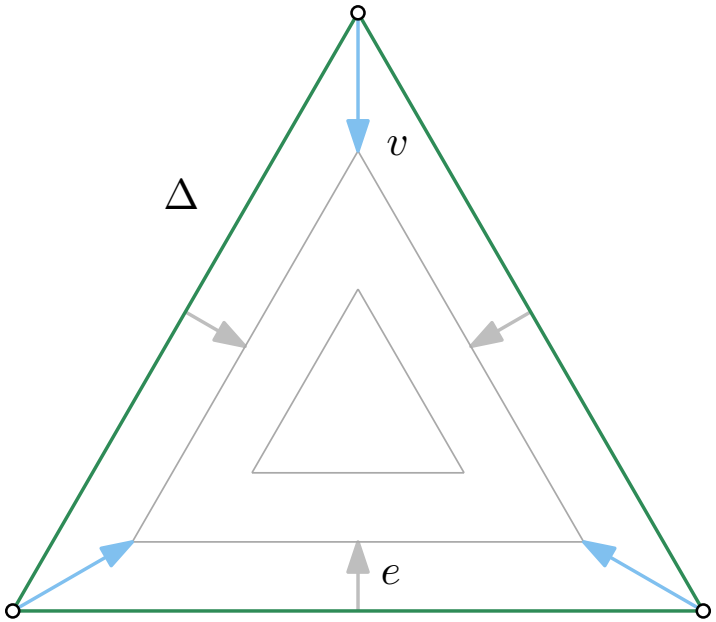
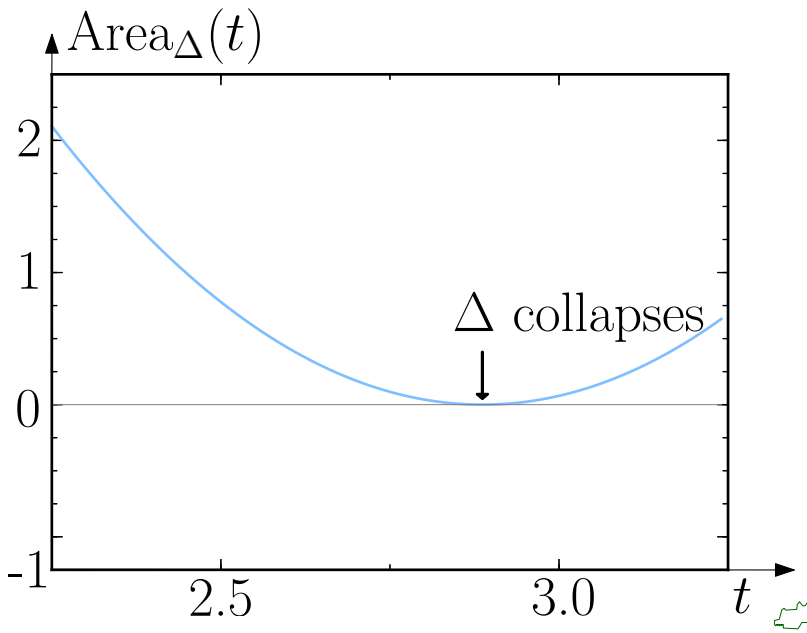
Implementation: Finding and Classifying Collapse Times

Different ways to compute collapse time

Suppose that the three vertices of a triangle move towards one point.

- The parabola plotted in blue is the (signed) area of the triangle over time, e.g., as obtained by means of determinant computations.



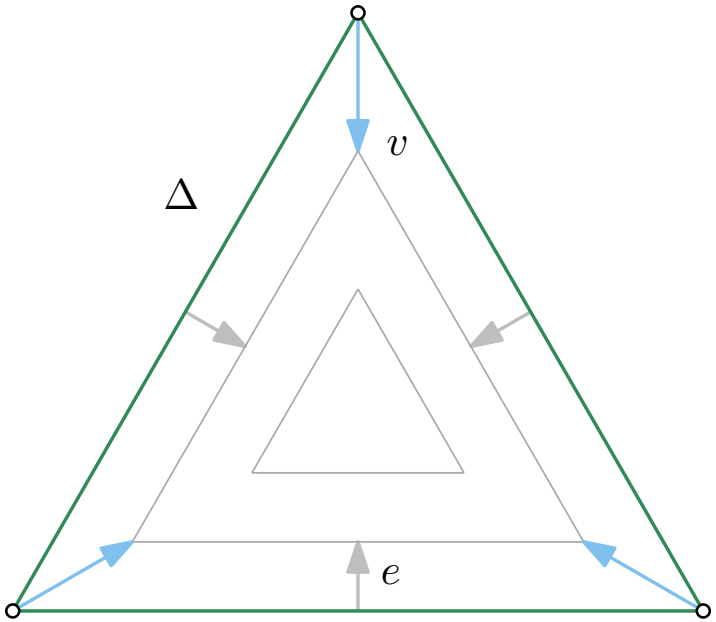


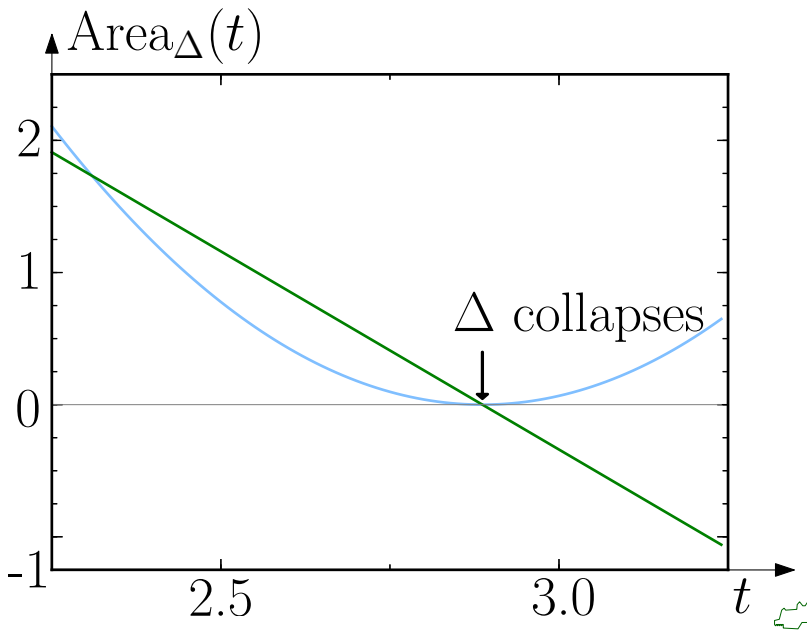
Implementation: Finding and Classifying Collapse Times

Different ways to compute collapse time

Suppose that the three vertices of a triangle move towards one point.

- The parabola plotted in blue is the (signed) area of the triangle over time, e.g., as obtained by means of determinant computations.
- The function in green represents the (signed) distance of one vertex to its opposite edge.





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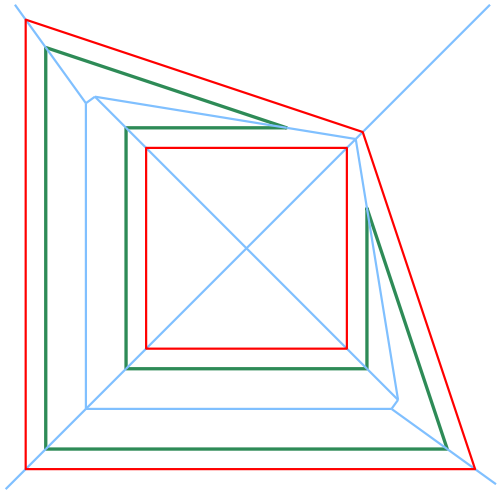

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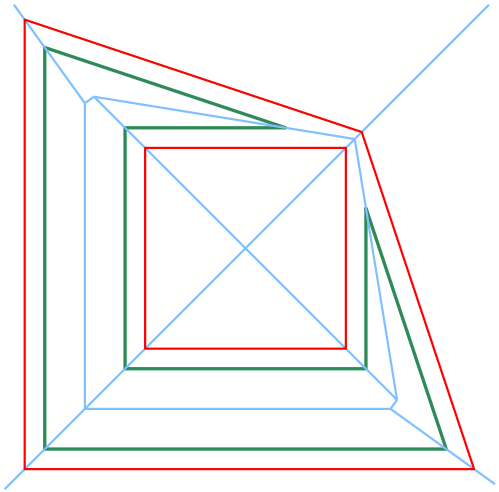

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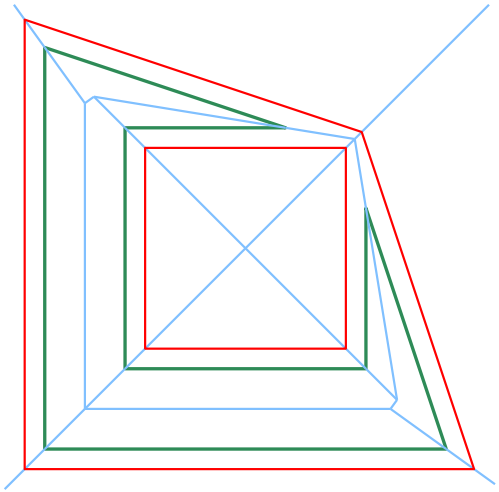

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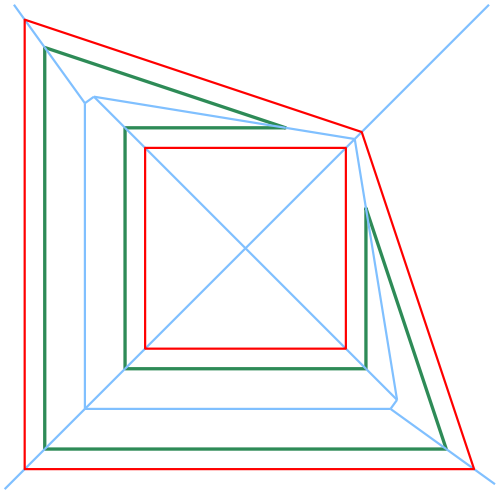
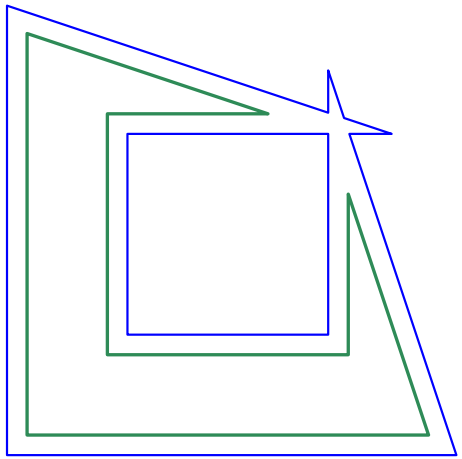
- CLIPPER and GEOS: Polygon-clipping libraries that apply general-purpose Boolean clipping algorithms to compute offsets.





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Experiments

- Simple closed polygons as test data.
- Input complexity n on x-axis, running time in seconds on y-axis.

Computation of one offset

- CLIPPER,
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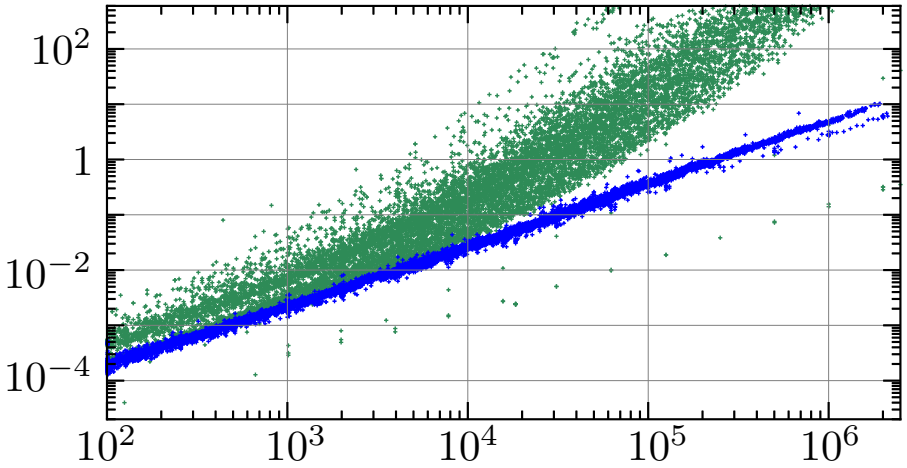

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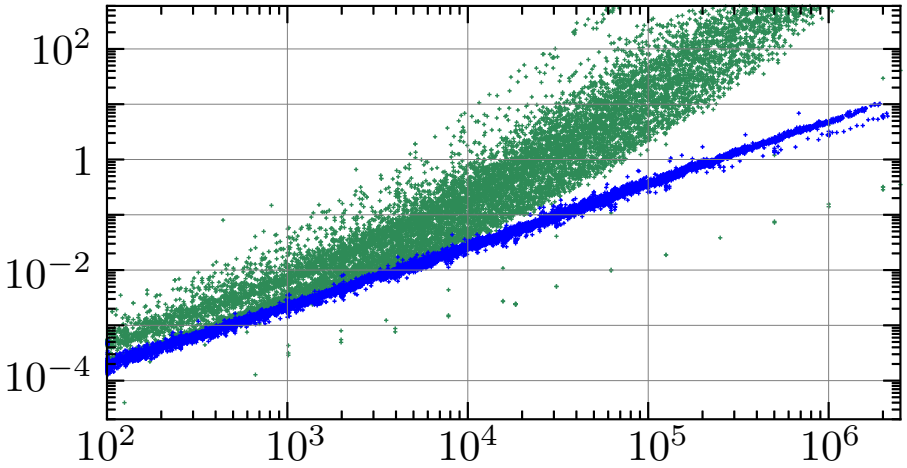
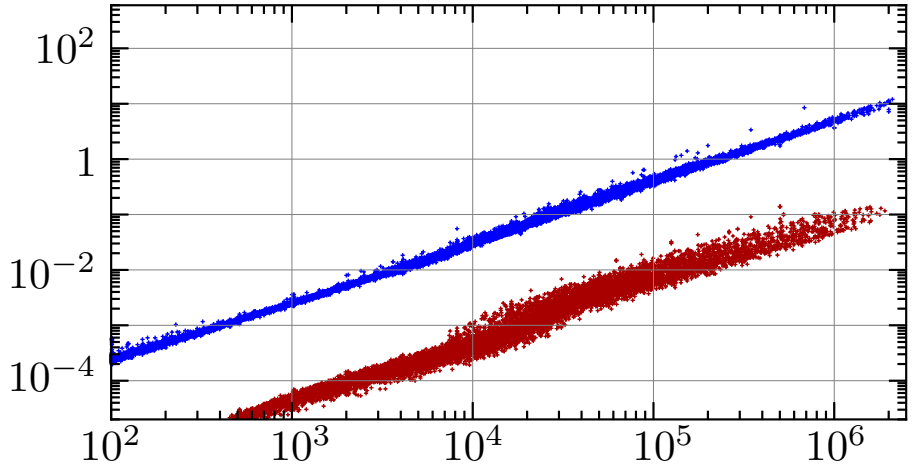


SK and SK-based offsetting

- Full SK by SURFER,
- One offset based on SK.



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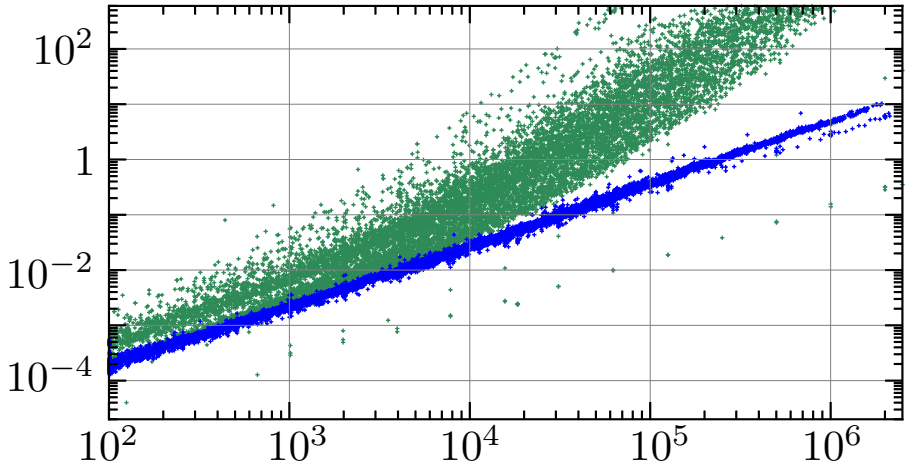

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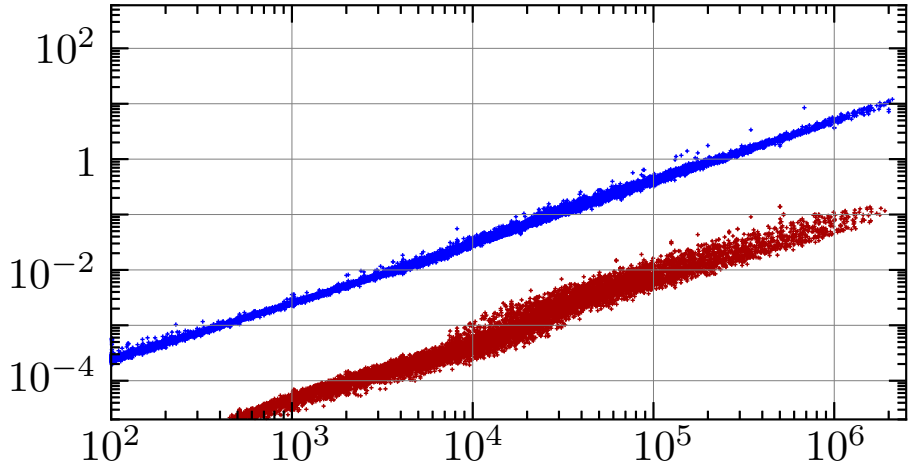
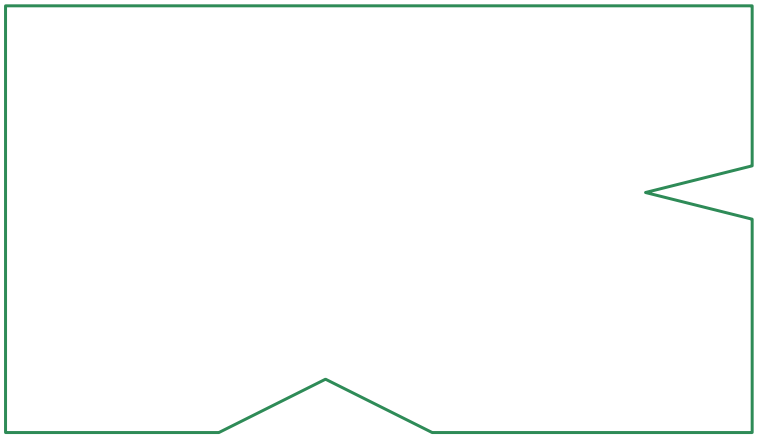


Experimental result

SURFER consumes roughly $5.8 \cdot 10^{-7} n \log n$ microseconds for an n-segment input. Except for a few convex polygons, a full run of SURFER is always (substantially) faster than the computation of one offset by CLIPPER.

Comparison of Sample Offsets



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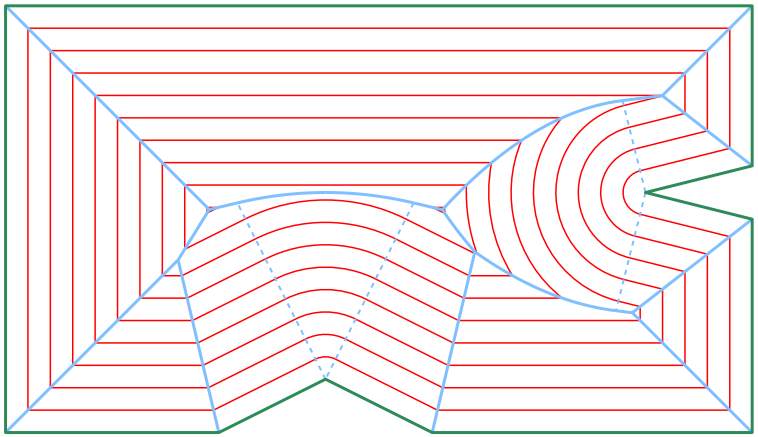

Voronoi diagram and rounded offsets

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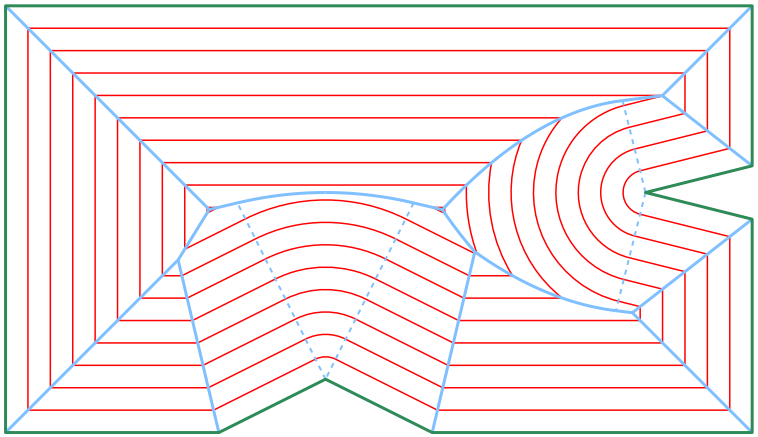

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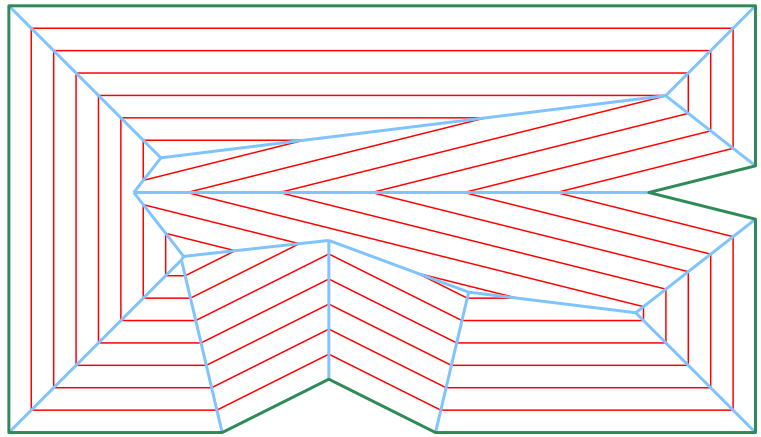

Straight skeleton and mitered offsets

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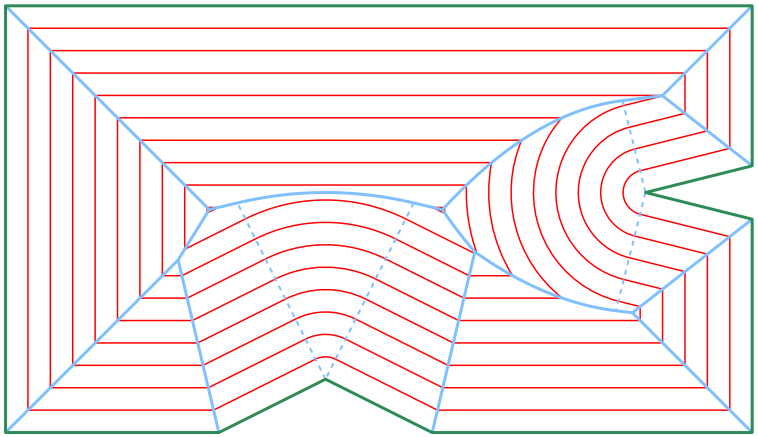

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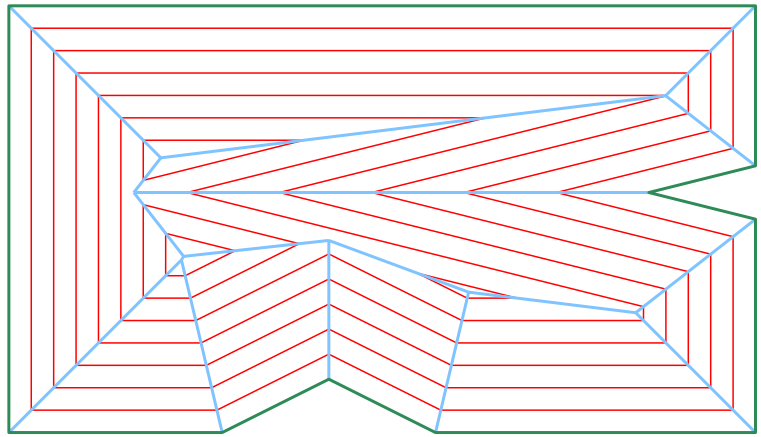

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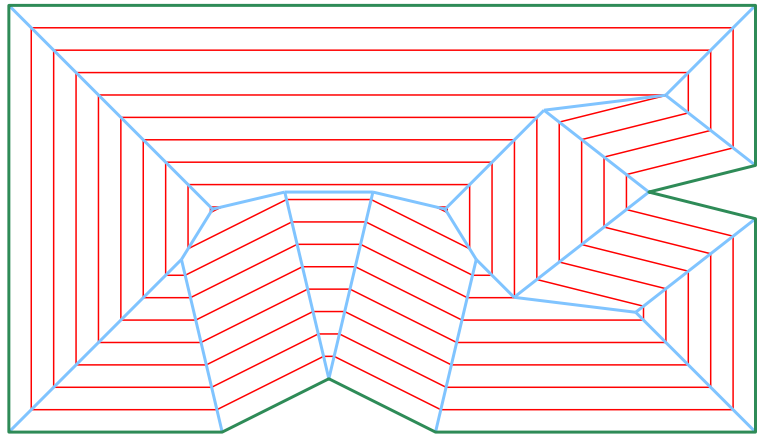
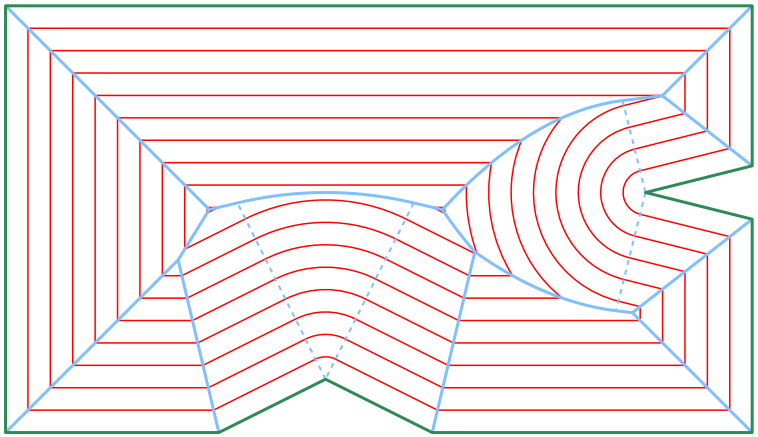


Straight skeleton and beveled offsets

Comparison of Sample Offsets



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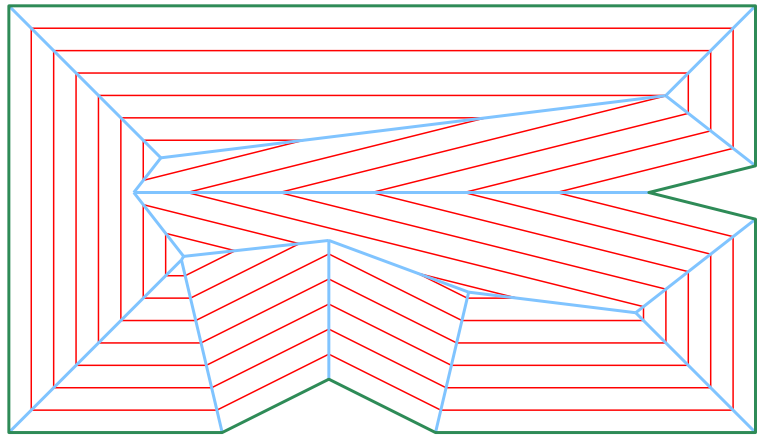

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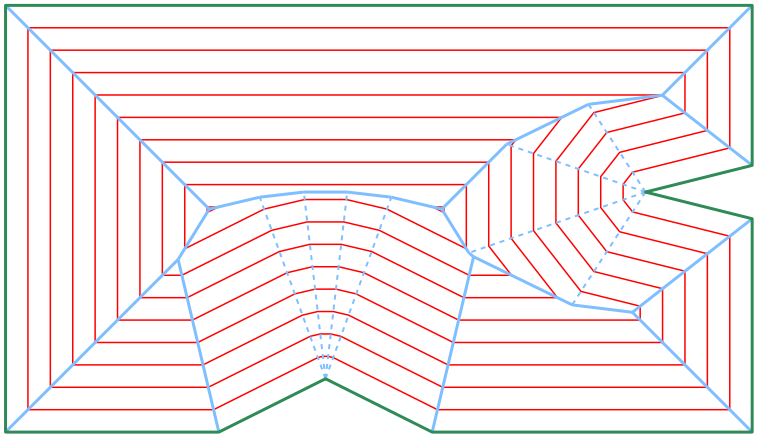
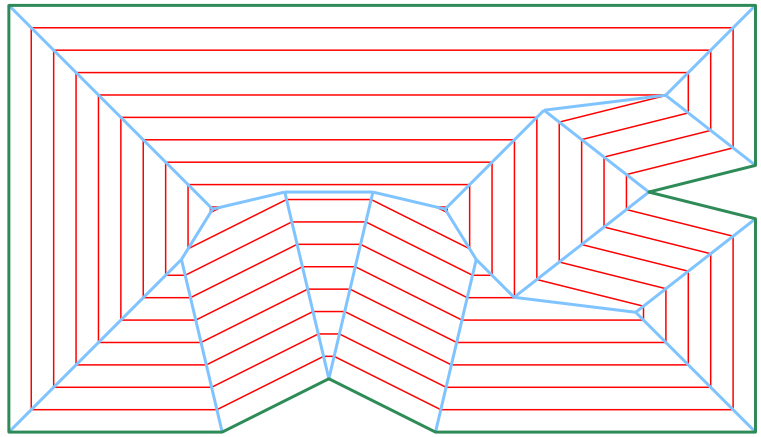


Linear axis and multi-segment bevels



Straight skeleton and beveled offsets

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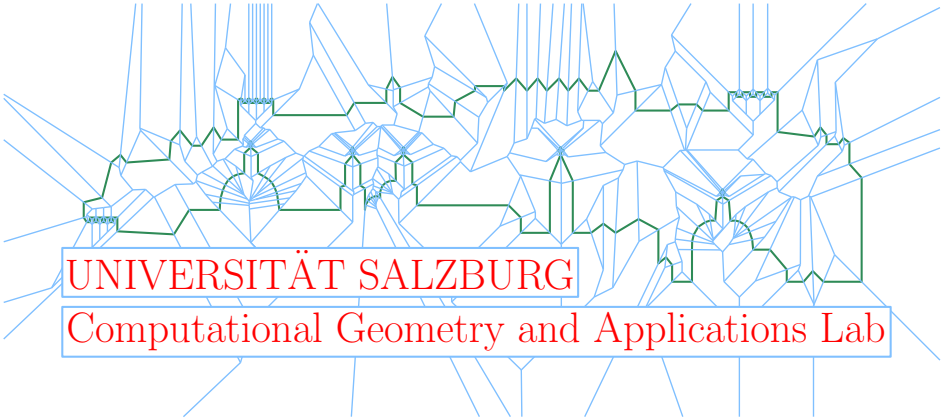
Gallery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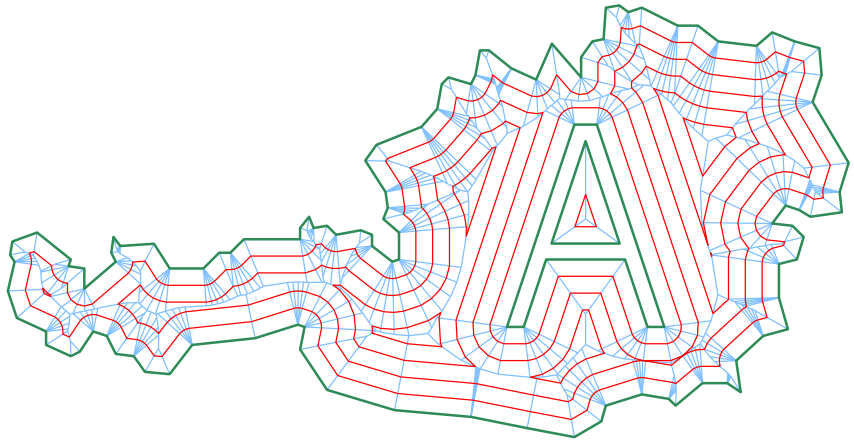

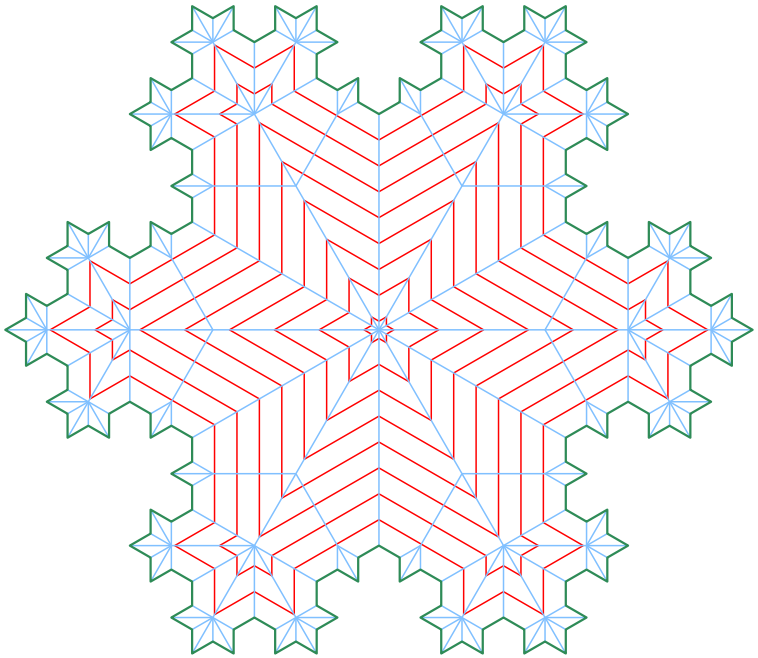

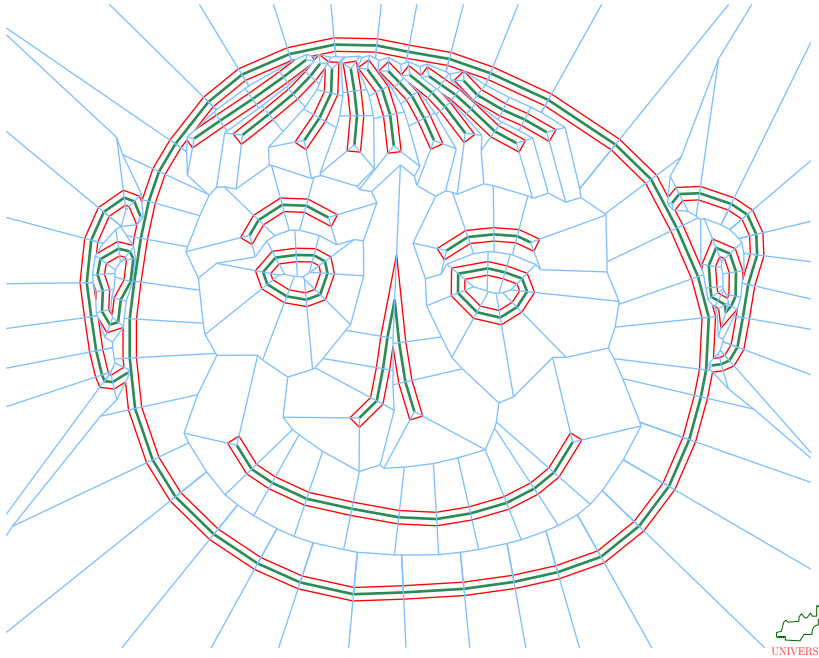

Thanks for Your Attention — Questions Welcome!

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